Sparse Subarray Design for Multitask Receivers

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***Abstract*—We consider sparse subarray design for multimission radars. The sparse array configuration significantly affects adaptive beamformer performance measured by the output signal-to-interference and noise ratio (SINR). In this paper, the full array is divided into two sparse subarrays, each performs a separate task. When antennas are not shared between the two subarrays due to differences in their properties and patterns, the design of two sparse subarray configurations to maximize the joint SINR must allocate each antenna position to one of the two tasks. We use Taylor series approximation to reformulate the underlying non-convex problem to a sequential convex programming (SCP) problem. The simulation results include examples of highly and weakly source spatial correlations, and they validate the efficiency of the proposed method.**

I. Introduction

The use of a sensor array has long been an attractive solution for statistical inference tasks, such as detection, estimation, tracking and filtering, in many applications, including radar, sonar, communication, satellite navigation, ultrasound, radio telescopes and seismology [1]–[3]. Adaptive beamforming utilizes sensor arrays to enhance the desired signals while mitigating interference and noise of the receiving array [4], [5]. The minimum variance distortionless response (MVDR) provides a unit response towards the desired direction and simultaneously reduces the noise and interference power at the array output [6]. Although the nominal array configuration is uniform, sparse arrays have recently emerged to play a fundamental role in various sensing systems involving multi-antenna transmitters and receivers. Diverse metrics have been proposed to design optimum sparse arrays in different applications such as direction finding, beampattern synthesis, target detection and spatial filtering [7]–[9]. Optimum sparse array design for maximizing SINR or signal-to-noise ratio (SNR) has been recently developed [10], [11] and shown to provide superior performance over structured sparse arrays that include uniform, nested, coprime, and minimum redundancy arrays. As shown in [12], [13], sparse array configuration has a significant effect on the performance of adaptive beamformers, including the MVDR beamformer.

Existing sparse array design techniques for maximizing SINR focus on a single task array where all available antennas are employed by a single beamforming. Their aim is to maximize the SINR at the receiver by properly configuring the array and determining the beamforming weights. However, situations may arise in active and passive sensing where the array is tasked with multiple missions, requiring two or more simultaneous beamformers. In this case, we are faced with the problem of multitask sparse subarray design. These tasks could belong to the same functionality, i.e., radar or communications or across different functionality as part of platform co-existence. Either case may demand unshared antennas among the subarrays to avoid compounded signal transmission and reception [14]. Furthermore, different tasks may warrant antennas with different patterns, properties, bandwidths, or polarizations, beside logistics and mutual coupling may prevent placing multiple antennas at the same location or in very close proximity - a concept known as shared aperture [15]. The same concept was discussed in [16] in which an algorithm was designed for a shared dual-band transmitting/receiving array antenna, where non-overlapping sparse subarrays of S- and X-band elements are designed on a single planar platform, constituting separate transmit and receive apertures for simultaneous transmitting and receiving operation. The mutual coupling of the adjacent antennas and the different type of antennas for each frequency band are also taken under consideration in sparse array design. These situations mandate that a permissible antenna placement grid point can accommodate only one antenna.

In this paper, we examine the scenarios of multimission or multitask sensing with sparse subarrays. In particular, we consider a uniform linear array (ULA) and two sources in the far field of the array, as shown in Fig.1. The source can be an active emitter or a target reflecting a transmitted waveform. The main goal of the proposed method is to simultaneously maximize the output SINR of the two sources, by optimally designing two sparse subarrays that collectively span the full length of the ULA. The optimum sparse subarray configurations are obtained by solving a joint SINR optimization applying MVDR beamforming. This optimization problem is non-convex, since the objective function of the maximization is not concave. We utilize Taylor series approximation of the objective function to render the optimization problem a convex one.

The rest of the paper is organized as follows: The system mathematical model is formulated in section II. The sparse subarray design is described in section III. Simulation results and comments upon the results are presented in section IV. The final remarks are given in section V.

II. System Model

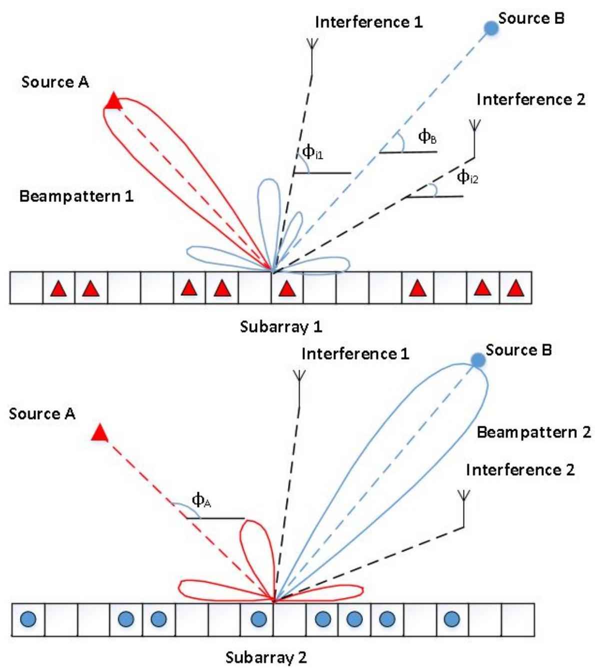

Fig. 1: System model with two interfering sources for $N = 16$.

We consider a ULA, consisting of N antennas with positions given by $y_n d$, $n = 1, \ldots, N$, where d represents the inter-element spacing. We assume that there are two sources at directions defined by ϕ_A and ϕ_B as viewed from the linear array. The primary objective of this work is to jointly design two sparse subarrays in order to maximize the SINR for both sources. This criterion translates into improved source or emitter detection performance. In either case, the source signals are assumed uncorrelated. Let K be the number of antennas in subarray A with coordinates $y_{An}d$, $n = 1, \ldots, K$, and the rest, $N - K$ antennas, form the second subarray B, placed at $y_{Bn}d$, $n = 1, \ldots, N - K$. There are m interfering signals impinging on the composite array from angles $\{\phi_{i1}, \ldots, \phi_{im}\}$. In the problem formulation, we consider source B acts as interference for subarray A and vice versa. An illustration of the system is presented in Fig.1. The corresponding steering vectors for each subarray towards direction ϕ can be written as:

$$\mathbf{a}(\phi) = [e^{jk_0 y_{A1} d cos\phi}, \ldots, e^{jk_0 y_{AK} d cos\phi}]^T \quad (1)$$

$$\mathbf{b}(\phi) = [e^{jk_0 y_{B1} d cos\phi}, \ldots, e^{jk_0 y_{B(N-K)} d cos\phi}]^T,$$

respectively, where k_0 is the wavenumber and is defined as $k_0 = 2\pi/\lambda$ with λ denoting the wavelength. The received signals for each subarray at time instant t are given by:

$$\mathbf{x}_A(t) = s_A(t)\mathbf{a}(\phi_A) + \mathbf{C}_A\mathbf{c}_A(t) + \mathbf{n}_A(t) \quad (2)$$

$$\mathbf{x}_B(t) = s_B(t)\mathbf{b}(\phi_B) + \mathbf{C}_B\mathbf{c}_B(t) + \mathbf{n}_B(t), \quad (3)$$

where $\mathbf{C}_A = [\mathbf{a}(\phi_B), \mathbf{a}(\phi_{i1}), \ldots, \mathbf{a}(\phi_{im})]$ and $\mathbf{C}_B = [\mathbf{b}(\phi_A), \mathbf{b}(\phi_{i1}), \ldots, \mathbf{b}(\phi_{im})]$ are the interference array manifold matrices with full column rank for subarrays A and B, respectively. The source signals are denoted as $s_A(t) \in \mathbb{C}$ and $s_B(t) \in \mathbb{C}$, respectively, with corresponding powers σ^2_{As} and σ^2_{Bs}. The vectors $\mathbf{c}_A(t) = [s_B(t), c_1(t), \ldots, c_m(t)]^T \in \mathbb{C}^{m+1}$ and $\mathbf{c}_B(t) = [s_A(t), c_1(t), \ldots, c_m(t)]^T \in \mathbb{C}^{m+1}$ represent the interfering signals for subarrays A and B, respectively, with covariance matrices $\mathbf{R}_{bA}$ and $\mathbf{R}_{bB}$, and $\mathbf{n}_A(t) \in \mathbb{C}^K$, $\mathbf{n}_B(t) \in \mathbb{C}^{N-K}$ denote the received Gaussian noise vectors at subarrays A and B with common power σ_n^2. The interference plus noise covariance matrices for subarrays A and B are defined as $\mathbf{R}_{nA} = \mathbf{C}_A\mathbf{R}_{bA}\mathbf{C}_A^H + \sigma_n^2\mathbf{I}_K$ and $\mathbf{R}_{nB} = \mathbf{C}_B\mathbf{R}_{bB}\mathbf{C}_B^H + \sigma_n^2\mathbf{I}_{N-K}$, respectively.

The received signal at subarray A is filtered by the K-length complex weight vector of subarray A denoted as $\mathbf{w}_A$. Thus, the output of the beamformer at subarray A is $\mathbf{w}_A\mathbf{x}_A(t)$, and the output SINR associated with source A is given by

$$\text{SINR}_A = \frac{\sigma^2_{As}|\mathbf{w}_A^H\mathbf{a}(\phi_A)|^2}{\mathbf{w}_A^H\mathbf{R}_{nA}\mathbf{w}_A}. \quad (4)$$

It is evident that, in order to maximize the SINR, the desired source signal must be secured while the undesired interference is suppressed or significantly mitigated. The MVDR beamformer minimizes the interference plus noise power at the beamformer output, subject to a distortionless response towards the direction of the desired source and is given by:

$$\mathbf{w}_A = \frac{\mathbf{R}_{nA}^{-1}\mathbf{a}(\phi_A)}{\mathbf{a}(\phi_A)^H\mathbf{R}_{nA}^{-1}\mathbf{a}(\phi_A)} \quad (5)$$

By substituting (5) into (4), we obtain the output SINR of the matched MVDR beamformer regarding source A as:

$$\text{SINR}_{oA} = \sigma^2_{As}\mathbf{a}(\phi_A)^H\mathbf{R}_{nA}^{-1}\mathbf{a}(\phi_A). \quad (6)$$

In order to present the full extent of the effect of the array configuration on the output SINR, we exploit the matrix inversion lemma and rewrite the interference plus noise covariance matrix $\mathbf{R}_{nA}^{-1}$ as:

$$\mathbf{R}_{nA}^{-1} = \sigma_n^{-2}[\mathbf{I}_K - \mathbf{C}_A(\mathbf{R}_{mA} + \mathbf{C}_A^H\mathbf{C}_A)^{-1}\mathbf{C}_A^H] \quad (7)$$

where $\mathbf{R}_{mA} = \sigma_n^2\mathbf{R}_{bA}^{-1}$. By defining $\text{SNR}_{iA} = \sigma^2_{As}/\sigma^2_n$ as the input signal-to-noise ratio (SNR) at subarray A and substituting (7) into (6), the output SINR at subarray A can be written as in (8).

By following the same steps for subarray B, the output SINR is given by (9), where $\text{SNR}_{iB} = \sigma^2_{Bs}/\sigma^2_n$ defines the SNR at subarray B and $\mathbf{R}_{mB} = \sigma_n^2\mathbf{R}_{bA}^{-1}$. It is evident from (8) and (9) that the output SINR at both subarrays is dependent on the array configuration through the source steering vectors $\mathbf{a}(\phi_A)$ and $\mathbf{b}(\phi_B)$ and the interference array manifold matrices $\mathbf{C}_A$ and $\mathbf{C}_B$.

III. Subarray Selection for matched MVDR beamforming

We consider a setting of N candidate linear, uniform grid locations, each equipped with an antenna. The respective

$$\mathrm{SINR}_{oA} = \mathrm{SNR}_{iA}[K - \mathbf{a}(\phi_A)^H \mathbf{C}_A (\mathbf{R}_{mA} + \mathbf{C}_A^H \mathbf{C}_A)^{-1} \mathbf{C}_A^H \mathbf{a}(\phi_A)] \tag{8}$$

$$\mathrm{SINR}_{oB} = \mathrm{SNR}_{iB}[(N-K) - \mathbf{b}(\phi_B)^H \mathbf{C}_B (\mathbf{R}_{mB} + \mathbf{C}_B^H \mathbf{C}_B)^{-1} \mathbf{C}_B^H \mathbf{b}(\phi_B)] \tag{9}$$

$$\begin{aligned} \max_{\mathbf{z}} \quad & \log|\hat{\mathbf{C}}_{aA}^H \mathcal{D}(\mathbf{z}) \hat{\mathbf{C}}_{aA} + \mathbf{R}_A| - \log|\hat{\mathbf{C}}_A^H \mathcal{D}(\mathbf{z}) \hat{\mathbf{C}}_A + \mathbf{R}_{mA}| + \\ & + \log|\hat{\mathbf{C}}_{aB}^H \mathcal{D}(\mathbf{1}_N - \mathbf{z}) \hat{\mathbf{C}}_{aB} + \mathbf{R}_B| - \log|\hat{\mathbf{C}}_B^H \mathcal{D}(\mathbf{1}_N - \mathbf{z}) \hat{\mathbf{C}}_B + \mathbf{R}_{mB}| \\ & s.t. \quad \mathbf{1}_N^T \mathbf{z} = K, \quad 0 \leq \mathbf{z} \leq 1 \end{aligned} \tag{10}$$

$$\log|\hat{\mathbf{C}}_A^H \mathcal{D}(\mathbf{z}) \hat{\mathbf{C}}_A + \mathbf{R}_{mA}| \approx \log|\hat{\mathbf{C}}_A^H \mathcal{D}(\mathbf{z}^{(k)}) \hat{\mathbf{C}}_A + \mathbf{R}_{mA}| + \bigtriangledown \mathbf{g}_A^T(\mathbf{z}^{(k)})(\mathbf{z} - \mathbf{z}^{(k)}) \tag{11}$$

$$\log|\hat{\mathbf{C}}_B^H \mathcal{D}(\mathbf{1}_N - \mathbf{z}) \hat{\mathbf{C}}_B + \mathbf{R}_{mB}| \approx \log|\hat{\mathbf{C}}_B^H \mathcal{D}(\mathbf{1}_N - \mathbf{z}^{(k)}) \hat{\mathbf{C}}_B + \mathbf{R}_{mB}| + \bigtriangledown \mathbf{g}_B^T(\mathbf{z}^{(k)})((\mathbf{1}_N - \mathbf{z}) - (\mathbf{1}_N - \mathbf{z}^{(k)})) \tag{12}$$

$$\begin{aligned} \max_{\mathbf{z}} \quad & \log|\hat{\mathbf{C}}_{aA}^H \mathcal{D}(\mathbf{z}) \hat{\mathbf{C}}_{aA} + \mathbf{R}_A| - \bigtriangledown \mathbf{g}_A^T(\mathbf{z}^{(k)})(\mathbf{z} - \mathbf{z}^{(k)}) + \log|\hat{\mathbf{C}}_{aB}^H \mathcal{D}(\mathbf{1}_N - \mathbf{z}) \hat{\mathbf{C}}_{aB} + \mathbf{R}_B| - \bigtriangledown \mathbf{g}_B^T(\mathbf{z}^{(k)})((\mathbf{1}_N - \mathbf{z}) - (\mathbf{1}_N - \mathbf{z}^{(k)})) \\ & s.t. \quad \mathbf{1}_N^T \mathbf{z} = K, \quad 0 \leq \mathbf{z} \leq 1 \end{aligned} \tag{13}$$

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receive beamforming weights are determined by utilizing MVDR beamforming, as presented in the previous section. We divide the receive array into two sparse subarrays which collectively span the N uniform array. Each subarray is concerned with one of the two sources. The optimum sparse subarray design towards sources A and B can be described as selecting the optimal K and $N-K$ candidate antennas that jointly maximize the SINR performance. Towards this goal, we define an antenna selection vector $\mathbf{z} \in \{0,1\}^N$, where entry "1" denotes an antenna selected for subarray A and a zero "0" entry denotes an antenna selected for subarray B. The diagonal matrix $\mathcal{D}(\mathbf{z})$ is the antenna selection operator with $\mathbf{z}$ populating the diagonal elements. Since we assume knowledge regarding all the antenna locations, the full array steering vector corresponding to direction ϕ is defined as $\hat{\mathbf{a}}(\phi) = [e^{jk_0 y_1 d cos\phi}, \dots, e^{jk_0 y_N d cos\phi}]^T$. Hence, the respective steering vectors for subarrays A and B towards direction ϕ can be expressed as $\mathbf{a}(\phi) = \mathbf{z} \odot \hat{\mathbf{a}}(\phi)$ and $\mathbf{b}(\phi) = (\mathbf{1}_N - \mathbf{z}) \odot \hat{\mathbf{a}}(\phi)$ and discard the zero entries, where $\mathbf{1}_N$ is an all one vector of size N and $\odot$ stands for the Hadamard product. In order to design the optimal sparse subarrays A and B, we consider the joint output SINR optimization problem as:

$$\begin{aligned} \max_{\mathbf{z}} \quad & \mathrm{SINR}_{oA} + \mathrm{SINR}_{oB} \\ s.t. \quad & \mathbf{1}_N^T \mathbf{z} = K, \quad 0 \leq \mathbf{z} \leq 1 \end{aligned} \tag{14}$$

Following [17], we can rewrite (14) as to maximize the sum of the logarithms of the output SINRs for both sources as in (10), where $\hat{\mathbf{C}}_{aA} = [\hat{\mathbf{C}}_A, \hat{\mathbf{a}}(\phi_A)]$ and $\hat{\mathbf{C}}_A = [\hat{\mathbf{a}}(\phi_B), \hat{\mathbf{a}}(\phi_{i1}), \dots, \hat{\mathbf{a}}(\phi_{im})]$. Similarly, $\hat{\mathbf{C}}_{aB} = [\hat{\mathbf{C}}_B, \hat{\mathbf{a}}(\phi_B)]$ and $\hat{\mathbf{C}}_B = [\hat{\mathbf{a}}(\phi_A), \hat{\mathbf{a}}(\phi_{i1}), \dots, \hat{\mathbf{a}}(\phi_{im})]$ and

$$\mathbf{R}_A = \begin{bmatrix} \mathbf{R}_{mA} & \mathbf{0}_{1\times m} \\ \mathbf{0}_{m\times 1} & 0 \end{bmatrix}, \quad \mathbf{R}_B = \begin{bmatrix} \mathbf{R}_{mB} & \mathbf{0}_{1\times m} \\ \mathbf{0}_{m\times 1} & 0 \end{bmatrix}.$$

It is clear that the binary constraint enforced by the antenna selection vector $\mathbf{z} \in \{0,1\}^N$ is not a convex constraint and it would render the optimization problem (10) non-convex. Hence, we replace the binary constraint with the box constraint $0 \leq \mathbf{z} \leq 1$, since the objective function of (10) can be written as the difference of two concave functions and the respective global optimizer locates at the extreme points of the polyhedron [18], [19]. However, (10) remains non-convex since the objective function is a difference of two concave functions. To overcome this problem, we utilize first order Taylor series that can iteratively approximate the second and fourth logarithm terms of the objective function, which cause the non-concavity of the objective function. The $(k+1)_{th}$ Taylor approximations of those terms based on the previous solution $\mathbf{z}^{(k)}$ are shown in (11) and (12), where $\bigtriangledown \mathbf{g}_A(\mathbf{z}^{(k)})$ and $\bigtriangledown \mathbf{g}_B(\mathbf{z}^{(k)})$ represent the gradients of the logarithmic functions $\log|\hat{\mathbf{C}}_A^H \mathcal{D}(\mathbf{z}) \hat{\mathbf{C}}_A + \mathbf{R}_{mA}|$ and $\log|\hat{\mathbf{C}}_B^H \mathcal{D}(\mathbf{1}_N - \mathbf{z}) \hat{\mathbf{C}}_B + \mathbf{R}_{mB}|$ evaluated at $\mathbf{z}^{(k)}$, respectively. That is,

$$\bigtriangledown \mathbf{g}_A = [\hat{\mathbf{d}}_{A,j}^H (\hat{\mathbf{C}}_A^H \mathcal{D}(\mathbf{z}^{(k)}) \hat{\mathbf{C}}_A + \mathbf{R}_{mA})^{-1} \hat{\mathbf{d}}_{A,j}, j = 1, ..., N]^T$$

where $\hat{\mathbf{d}}_{A,j}$ denotes the j_{th} column vector of the matrix $\hat{\mathbf{C}}_A^H$. The gradient for subarray B is analogously defined. By utilizing sequential convex programming (SCP), the initially non-convex problem is reformulated to a series of convex subproblems, that can be optimally solved via interior point methods [20]. By substituting (11) and (12) in (10), we obtain the approximated convex optimization problem as shown in (13). It should be noted that the first term at the right side of the equations (11) and (12) are omitted from the optimization (13), since they are constants and do not affect the result of the optimization. Since SCP is a local heuristic, the solution of (13) is dependent on the selection of the initial $\mathbf{z}^{(0)}$. Therefore, we initialize the SCP algorithm with a number of feasible vectors $\mathbf{z}^{(0)}$ and keep the solution that provides the maximum objective function value.

IV. Simulation Results

In this section, simulation results are presented to validate the proposed optimal sparse subarray design for maximizing the output SINR for two different sources. We consider a uniform linear array (ULA) of $N = 16$ antennas with an inter-element spacing of $d = \lambda/2$. We assume two subarrays of equal number of antennas, $K = 8$, and each subarray is designated to one source. The first source signal is incident on the array with direction ϕ_A that is shifting from 0^o to 180^o with a step of 5^o, and with an SNR set at 0dB. The second source is fixed at $\phi_B = 60^0$ with an SNR set at 5dB. We design the optimum sparse subarrays A and B for each ϕ_A using (13). Subarray A considers source B as an interference and vice versa. No other interferences are injected. In order to validate the efficiency of the Taylor series approximation SCP, Fig.2 shows the comparison of the output SINR from (13) for each subarray with the respective optimum output SINR through enumeration. It is evident that the proposed subarray selection iterative algorithm closely approximates the global optimum solution obtained from enumeration. There is also a notable drop at the output SINR of both sources when they are closely separated, since each source acts as an interfering signal for the processing of the other source. In order to display the effect of the subarray selection on the output SINR, we set $\phi_A = 100^o$ and enumerate every possible subarray configuration. Fig.3 presents the output SINR of subarrays A and B in a descending order with respect to all the 12870 different selections. It is evident that there is a substantial impact of different subarray configurations on the output SINR.

In the second example, we examine the performance of the proposed algorithm for high and weak spatially correlated sources. We extend the ULA to $N = 24$ antennas and set the number of sensors in each subarray to $K = 12$. For the first case of highly spatially correlated sources, the angles-of-arrival of the two sources are $\phi_A = 93^o$ and $\phi_B = 91^o$, respectively. In the second case, the source signals arrive from angles $\phi_A = 135^o$ and $\phi_B = 50^o$, respectively. The spatial correlation matrices of the source steering vectors corresponding to the high and the low correlation cases are:

$$\mathbf{R}_{high} = \begin{bmatrix} 1 + 0j & 0.2830 + 0.8819j \\ 0.2830 - 0.8819j & 1 + 0j \end{bmatrix},$$

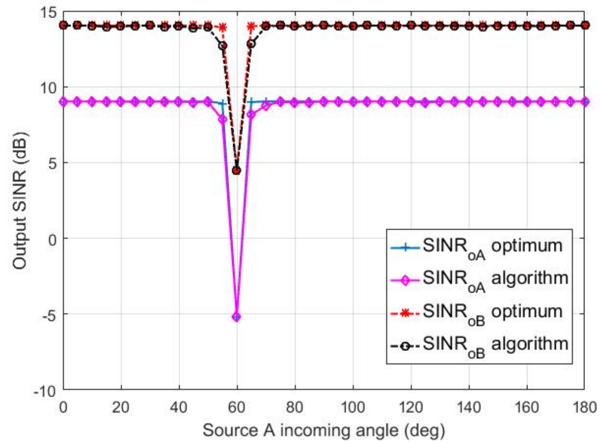

Fig. 2: Output SINR for subarrays A and B derived from enumeration and proposed method.

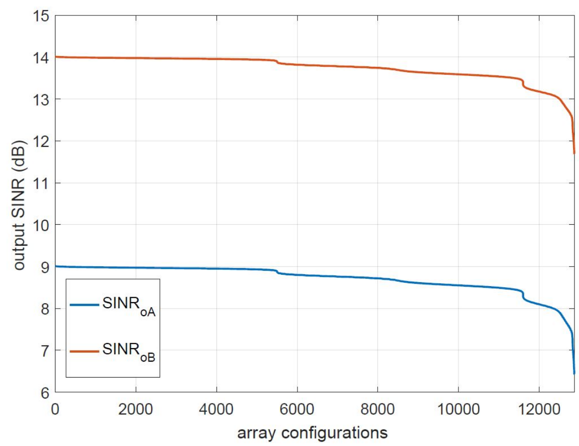

Fig. 3: Output SINR for all different sparse subarrays for $\phi_A = 100^o$ and $\phi_B = 60^0$.

$$\mathbf{R}_{low} = \begin{bmatrix} 1 + 0j & 0.0023 + 0.0303j \\ 0.0023 - 0.0303j & 1 + 0j \end{bmatrix}.$$

For both cases, the SNR of each source is set at 0dB. An additional interfering signal is considered in the model, impinging at the receiving array from $\phi_{i1} = 108^o$ with INR equal to 20dB. In order to shed light to the mechanism of joint optimal subarray design, we also derive the true optimal subarrays that maximize the output for each of the sources A and B separately, by solving the following optimization using SCP for both the high and the low correlation cases:

$$\max_{\mathbf{z}} \quad \text{SINR}_{oi} \quad s.t. \quad \mathbf{1}_N^T \mathbf{z} = K, \quad 0 \leq \mathbf{z} \leq 1 \tag{15}$$

for $i = A, B$. Figs.4 and 5 depict the optimum sparse arrays obtained from (15) and the optimum sparse subarrays obtained from the proposed method in (13) for the highly correlated and less correlated cases, respectively. It is observed that in the highly correlated case, the optimum sparse arrays for sources A and B obtained from separate design are almost fully-overlapped and consist of the same antennas except one,

TABLE I: Maximum SINR for the proposed method of (13) and separate optimization (15) (dB).

	Joint opt.(Eq.(13))	Separate opt.(Eq.(15))
$SINR_{oA}$, $\phi_A = 93^o$	7.5068	9.2781
$SINR_{oB}$, $\phi_B = 91^o$	7.9369	9.3065
$SINR_{oA}$, $\phi_B = 135^o$	10.7526	10.7743
$SINR_{oB}$, $\phi_B = 50^o$	10.7426	10.7730

whereas the optimum sparse arrays for the less correlated case share only 5 antennas. Therefore, the competition for the optimal antennas in the joint subarray selection is higher for highly correlated sources. As depicted in Fig.4, arrays (c) and (d) obtained from (13) opt to share the optimal antennas located at the two edges of the ULA and the rest of the antennas located at the center of the ULA, in order to maximize their output SINR. Table I presents the maximum output SINR values for optimizations (13) and (15) for the considered cases. For the case of highly correlated sources, the proposed method provides a significantly lower SINR as compared to the optimum SINR obtained from separate optimization, whereas for less correlated sources the joint design almost matches the separate optimization. The beampatterns for the subarrays obtained from (13) for the high and low spatial correlation cases are shown in Fig.6 and Fig.7, respectively.

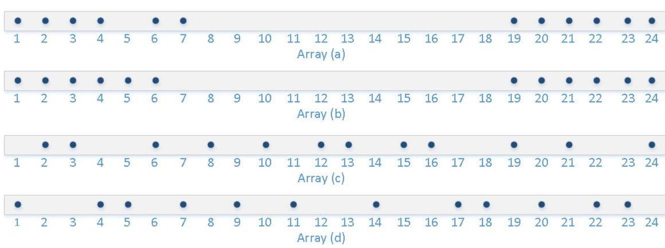

Fig. 4: Array: (a) separate design for $\phi_A = 93^o$, (b) separate design for $\phi_B = 91^o$, (c) joint optimization for $\phi_A = 93^o$, (d) joint optimization for $\phi_B = 91^o$

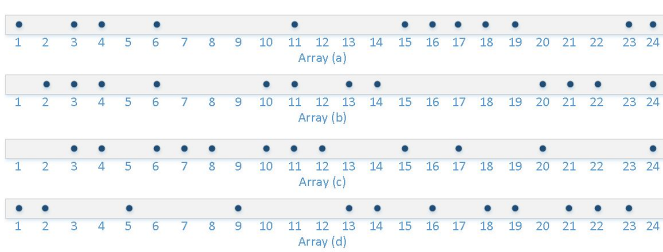

Fig. 5: Array: (a) separate design for $\phi_A = 135^o$, (b) separate design for $\phi_B = 50^o$, (c) joint optimization for $\phi_A = 135^o$, (d) joint optimization for $\phi_B = 50^o$

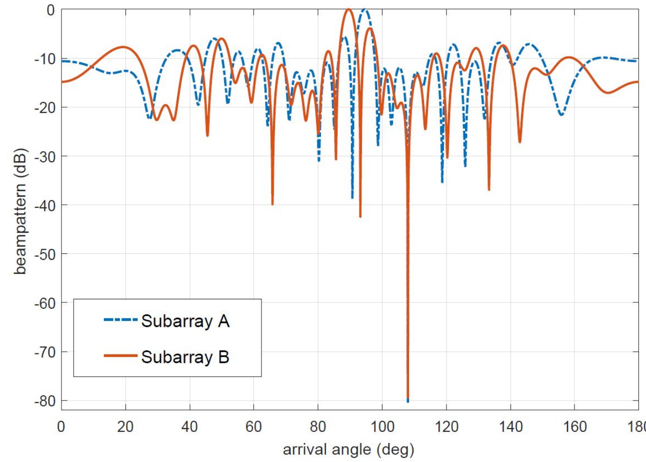

Fig. 6: The beampatterns for subarrays A and B for $\phi_A = 93^o$ and $\phi_B = 91^o$.

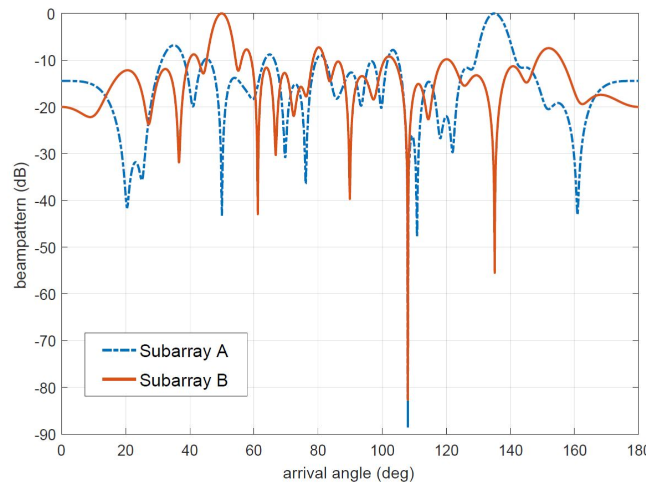

Fig. 7: The beampatterns for subarrays A and B for $\phi_A = 135^o$ and $\phi_B = 50^o$.

In order to test the efficiency of the proposed method we compare the results of (13) to the case when prefixed nested and coprime arrays are utilized to process source A [21], [22]. We assume that the direction of the first and the second sources as seen from the ULA is $\phi_A = 135^o$ and $\phi_B = 50^o$, respectively, but for this system we assume a ULA of $N = 16$ arrays, where $K = 7$ antennas are used to process source A. One interfereing source is considered at $\phi_{i1} = 108^o$ with

TABLE II: Maximum SINR for the proposed method, the nested arrays and the coprime arrays (dB).

	Proposed method	Nested	Coprime
$SINR_{oA}$, $\phi_A = 135^o$	8.4346	7.6046	6.1278
$SINR_{oB}$, $\phi_B = 50^o$	9.4791	9.1356	8.8276

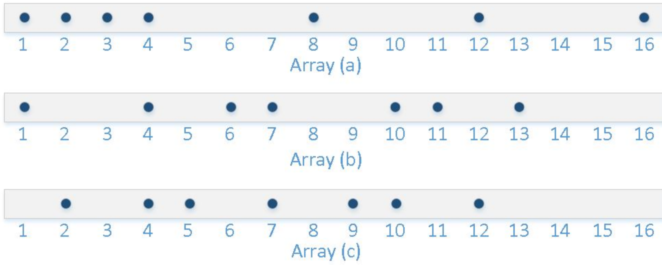

Fig. 8: Subarray A: (a) Nested array, (b) Coprime array, (c) proposed method (13).

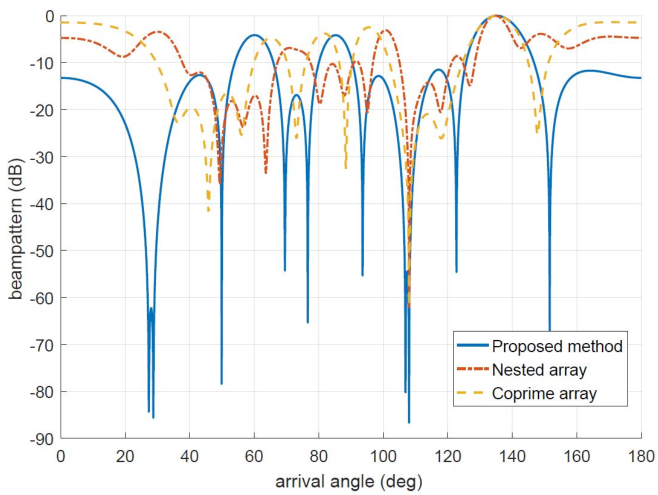

Fig. 9: Beampatterns for subarrays A in Fig.8.

INR equal to 20dB. The subarray A structures are given in Fig.8 and the corresponding beampaterns in Fig.9. As shown in Fig.9, the proposed joint subarray design yields a better shaped beampattern with lower sidelobes and deeper nulls at the direction of interference as compared to the nested array and coprime beampatterns. Table II shows the maximum SINR obtained from the proposed joint optimization method and the prefixed nested and coprime techniques. As expected the proposed adaptive technique significantly outperforms the predefined nested and coprime arrays in terms of output SINR for both sources A and B.

V. Conclusion

We presented a sparse subarray design technique for multitask receivers. The main contribution of this method is the joint design of two sparse subarrays; each handles one beam or one source. The selection of the optimal sparse subarrays is decided by performing a joint SINR maximization for matched MVDR beamforming. The simulation results confirmed the efficiency of the algorithm and the superiority of the proposed adaptive technique over prefixed subarray selection.

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References

[1] L. E. Brennan and L. S. Reed, "Theory of adaptive radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-9, pp. 237–252, 1973.

[2] S. Applebaum, "Adaptive arrays," *IEEE Transactions on Antennas and Propagation*, vol. 24, no. 5, pp. 585–598, 1976.

[3] P. Napier, A. Thompson, and R. Ekers, "The very large array: Design and performance of a modern synthesize radio telescope," *Proceedings of the IEEE*, vol. 71, no. 11, pp. 1295–1320, 1983.

[4] A. Deligiannis, S. Lambotharan, and J. A. Chambers, "Beamforming for fully-overlapped two-dimensional Phased-MIMO radar," *IEEE Radar Conference (RadarCon), Arlington, VA, USA, 2015.*

[5] ——, "Game Theoretic Analysis for MIMO Radars With Multiple Targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 6, pp. 2760–2774, 2016.

[6] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, vol. 57, pp. 1408–1418, 1969.

[7] A. Bertrand, J. Szurley, P. Ruckebusch, I. Moerman, and M. Moonen, "Efficient calculation of sensor utility and sensor removal in wireless sensor networks for adaptive signal estimation and beamforming," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5857–5869, 2012.

[8] S. P. Chepuri and G. Leus, "Sparsity-promoting sensor selection for non-linear measurement models," *IEEE Transactions on Signal Processing*, vol. 63, no. 3, pp. 684–698, 2015.

[9] X. Wang, E. Aboutanios, and M. G. Amin, "Adaptive array thinning for enhanced DOA estimation," *IEEE Signal Processing Letters*, vol. 22, pp. 799–803, 2015.

[10] O. T. Demir and T. E. Tuncer, "Optimum discrete transmit beamformer design," *Digital Signal Processing*, vol. 36, pp. 57–68, 2015.

[11] X. Wang, M. G. Amin, X. Wang, and X. Cao, "Sparse array quiescent beamformer design combining adaptive and deterministic constraints," *IEEE Transactions on Antennas and Propagation*, vol. PP, no. 99, pp. 1–11, 2017.

[12] M. Wax and Y. Anu, "Performance analysis of the minimum variance beamformer," *IEEE Transactions on Signal Processing*, vol. 44, no. 4, pp. 928–937, 1996.

[13] M. G. Amin, X. Wang, Y. D. Zhang, F. Ahmad, and E. Aboutanios, "Sparse arrays and sampling for interference mitigation and DOA estimation in GNSS," *Proceedings of the IEEE*, vol. 104, no. 6, pp. 1302–1317, 2016.

[14] A. Hassanien, M. W. Morency, A. Khabbazibasmenj, S. A. Vorobyov, J.-Y. Park, and S.-J. Kim, "Two-dimensional transmit beamforming for MIMO radar with sparse symmetric arrays," *IEEE Radar Conference (RADAR)*, 2013.

[15] D. M. Pozar and S. D. Targonski, "A shared-aperture dual-band dual-polarized microstrip array," *IEEE Transactions on Antennas and Propagation*, vol. 49, no. 2, pp. 150–157, 2001.

[16] G. Kwon, J. Y. Park, D. H. Kim, and K. C. Hwang, "Optimization of a Shared-Aperture Dual-Band Transmitting/Receiving Array Antenna for Radar Applications," *IEEE Transactions on Antennas and Propagation*, vol. XX, no. X, pp. 1–14, 2017.

[17] X. Wang, E. Aboutanios, and M. G. Amin, "Slow radar target detection in heterogeneous clutter using thinned space-time adaptive processing," *IET Radar, Sonar & Navigation*, vol. 10, no. 4, pp. 726–734, 2016.

[18] H. Tuy, *Convex analysis and global optimization*. Springer, 1998.

[19] R. Horst, *Introduction to global optimization*. Springer, 2000.

[20] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.

[21] P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4167–4181, 2010.

[22] M. G. Amin, P. P. Vaidyanathan, Y. D. Zhang, and P. Pal, "Special issue on coprime sampling and arrays," *Digital Signal Processing*, vol. 61, pp. 1–96, 2017.