

Multipath-Aware Velocity Estimation for Sparsity-Based Through-the-Wall Radar Imaging

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Abstract—Through-the-wall radar imaging utilizes electromagnetic wave propagation to reveal the locations and velocities of obscured targets. For a pulsed radar operation, conventional Doppler processing can only estimate the radial velocity component of the indoor moving targets. We show that multipath propagation provides additional Doppler information that allows estimation of the full target velocity vector. We propose a computationally efficient two-step approach that obtains the target locations using sparse reconstruction and estimates the velocities by utilizing the Doppler information in the various propagation paths. We support our findings by simulation and experimental results.

I. INTRODUCTION

Through-the-wall radar imaging (TWRI) has gained attention due to its ability to acquire accurate information of scenes behind walls or other opaque obstacles using electromagnetic (EM) wave propagation [1]–[4].

In many TWRI applications, it is desirable to determine both the locations and the velocities of the targets. Resolving the full target velocity vectors is usually not feasible in single-site radar deployment as only the velocity component perpendicular to the trajectory of constant range, i.e. the Doppler shift, is observable. However, multipath propagation can provide additional information about the targets' movements because the observed Doppler shift changes with the propagation path of the signal. As such, multipath exploitation can enable accurate resolution of the full velocity vectors by utilizing the Doppler diversity provided by the multipath target returns. Further, highly resolved scene reconstructions are desirable in TWRI applications, which require use of large apertures and bandwidth leading to huge amounts of acquired data. In order to tackle the data deluge, compressive sensing (CS) has been applied to TWRI [5]. Joint reconstruction of stationary and moving targets without consideration of multipath [6] and including multipath exploitation [7] have been addressed in terms of CS in TWRI. However, the latter is computationally very demanding.

In this paper, we propose a two-step approach to locate targets and estimate their velocities in an indoor multipath

environment. Our scheme is based on the efficient approach originally proposed in [8] for reconstruction of stationary and moving targets in a multipath free environment. We build on the aforementioned scheme by taking the multipath returns into account in the signal model. In the first step, we utilize a CS based method [9] to reconstruct a sparse image of the scene and to separate the returns from the various paths. This reconstruction is carried out individually for each pulse in the coherent processing interval (CPI). In the second step, Doppler processing of the localized targets is performed across the pulses for each path. Hence, we obtain as many Doppler velocities estimates for each target as the number of available propagation paths. Using least-squares estimation, we can then determine the full velocity vector for each target from the Doppler information contained in the various paths. We present analytic, simulation and experimental results to demonstrate the low computational complexity and the effectiveness of the proposed two-step approach.

The remainder of the paper is organized as follows. The signal model is introduced in Section II. In Section III, we describe the proposed method for efficient target localization and velocity estimation. Simulation and experimental results are presented in Section IV. Section V concludes the paper.

II. SIGNAL MODEL

In this section, we describe the signal model for an ultra-wideband multistatic pulsed radar system with a single transmitter and N receivers. For simplicity, we consider the single transmitter case; however, the model can be readily extended to the case of multiple transmitters with the assumption of time-multiplexed operation.

We assume P point targets located in a two-dimensional (2D) area of interest, with $\mathbf{x}_p = (x_p, y_p)$ being the position vector of the p th target. A wideband pulse, $s(t)$, in the complex baseband with carrier frequency f_c is transmitted. The pulse travels through the exterior wall to the P point targets and is reflected back to the receive array. The baseband radar return received by the n th receiver is given by

$$z_n(t) = \sum_{p=0}^{P-1} \sigma_p s(t - \tau_{pn}) \exp(-j2\pi f_c \tau_{pn}) \quad (1)$$

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where σ_p is the complex reflectivity of the p th point target and τ_{pn} is the bistatic propagation delay from the transmitter to the p th target and back to the n th receiver.

A discrete version of the signal model is generated by sampling the target space at N_p points, which can be stacked into a vector σ . Each grid point assumes a certain reflectivity, with the absence of a target at a particular grid point represented by zero value for the corresponding reflectivity. The signals $z_n(t)$, $n = 0, 1, \dots, N-1$, are also sampled uniformly at N_T time steps resulting in vectors z_n , $n = 0, 1, \dots, N-1$, each of length N_T . The resultant discretized linear measurement model can be expressed as

$$z = \Psi \sigma, \quad (2)$$

where z contains the stacked measurements z_n from the N receivers and Ψ is the dictionary obtained by discretizing the right hand side of (1).

At this point, we consider two extensions to the above linear measurement model. First, we consider slowly moving targets with linear motions. We assume that the motions are slow enough that the targets are approximately stationary during a single pulse. Also, the targets are assumed to stay within the same range cell over a CPI of K pulses. This leads to a change of the phases in the individual reflectivity vectors $\sigma(k)$ and, hence, to individual measurement vectors $z(k)$ for each pulse $k = 0, \dots, K-1$. Note that due to the slow motion assumption, the phase progression in $\sigma(k)$ is approximately linear for each target. Second, we introduce an additive multipath model as described in [9]. Each possible propagation path $r = 0, \dots, R-1$ leads to an additive component, yielding

$$z(k) = \Psi^{(0)} \sigma^{(0)}(k) + \Psi^{(1)} \sigma^{(1)}(k) + \dots + \Psi^{(R-1)} \sigma^{(R-1)}(k). \quad (3)$$

Note that $r = 0$ corresponds to the direct path and $r = 1, 2, \dots, R-1$ are the multipaths. The multipath dictionaries $\Psi^{(r)}$, $r = 1, 2, \dots, R-1$ account for the two-way delays due to indirect propagation from the transmitter to the target space and back to the receivers. Note that we assume an individual target state vector $\sigma^{(r)}(k)$ for each path as the phase and amplitude of the target reflectivity, in general, change with the bistatic and aspect angles. Further, an additional subsampling step, consistent with the nature of the CS framework, can be introduced in (3) to reduce the number of temporal and spatial measurements.

III. TWO-STEP LOCATION AND VELOCITY ESTIMATION

A. Group Sparse Location Estimation

Given the measurements $z(k)$ in (3) corresponding to the k th radar pulse, we first aim at recovering the target state vectors $\sigma^{(r)}(k)$ using sparse reconstruction. The vectorized scenes $\sigma^{(r)}(k)$, corresponding to the R paths, exhibit a group sparse structure, where the individual groups extend across the paths for each target location. Employing group sparse

reconstruction results in the optimization problem [9], [10]

$$\hat{\sigma}^{(r)}(k) = \arg \min_{\sigma^{(r)}(k), r=0, \dots, R-1} \left\| z(k) - \sum_{r=0}^{R-1} \Psi^{(r)} \sigma^{(r)}(k) \right\|_2^2 + \lambda \sum_{p=0}^{N_p-1} \left\| \left[\sigma_p^{(0)}(k), \sigma_p^{(1)}(k), \dots, \sigma_p^{(R-1)}(k) \right]^T \right\|_2, \quad (4)$$

where $k = 0, \dots, K-1$ and λ is a regularization parameter. The optimization problem in (4) can be solved using SparSA [11] or other available schemes [12], [13].

Essentially, group sparse reconstruction provides a scheme to associate the acquired target returns with the various propagation paths. Once the reflectivity vectors $\hat{\sigma}^{(r)}(k)$ are reconstructed by solving (4) for all $k = 0, \dots, K-1$, an intermediate image containing target location estimates can be formed by a non-coherent summation over all paths and pulses. At this point, a target detection step should be carried out to select only N_{detect} targets with significant amplitude. This keeps the computational complexity of the subsequent velocity estimation procedure as low as possible.

B. Multipath-Aware Velocity Estimation

The velocity vector for each target is estimated by further processing of the sparse reconstruction results. For each of the N_{detect} targets, we obtain KR complex amplitudes, one for every combination of propagation path and pulse, denoted by $b_p^{(r)} = [\hat{\sigma}_p^{(r)}(0), \dots, \hat{\sigma}_p^{(r)}(K-1)]^T \in \mathbb{C}^K$, $p = 0, \dots, N_{\text{detect}}-1$, $r = 0, \dots, R-1$. The Doppler velocity of the target causes a phase progression along the slow time dimension k . The amount of phase progression per pulse encodes the Doppler velocity and depends on the location/velocity of the target as well as the propagation path. We take the discrete-time Fourier transform of $b_p^{(r)}$ along the slow time to obtain the Doppler information $B_p^{(r)}(\omega)$ for the targets. Assuming only a single target per location cell, we can find the Doppler velocities for each target and path by finding the peaks in the Fourier-transformed slow time vectors. Hence, for each target, we obtain R Doppler velocities

$$v_{D,p}^{(r)} = \frac{c}{\pi f_c} \arg \max_{\omega} B_p^{(r)}(\omega) \quad (5)$$

that correspond to the projections onto the normal velocity vectors. The direction of the normal vector is perpendicular to the constant range trajectory of the respective target and path. This is the same concept as the radial velocity component in a monostatic radar.

The linear relationship between the target velocity vector and the Doppler velocities can be expressed as

$$\begin{bmatrix} v_{D,p}^{(0)}, \dots, v_{D,p}^{(R-1)} \end{bmatrix}^T = \mathbf{V}_p \begin{bmatrix} v_{px} \\ v_{py} \end{bmatrix}, p = 0, \dots, N_{\text{detect}}-1, \quad (6)$$

where the rows of \mathbf{V}_p contain the normal velocities for each path at the location of the p th target. The over-determined linear equation system (6) can be solved using a least squares approach to obtain a velocity estimate $(\hat{v}_{px}, \hat{v}_{py})^T$. Note that

the exploitation of multipath enables an estimate of the full target velocity vector, whereas a single path may only deliver a scalar Doppler speed.

The described velocity estimation method can be extended to multiple targets within a single location resolution cell. In this case, multiple Doppler velocities need to be extracted from each path. This, however, results in resolution and association issues. First, multiple Doppler velocities may only be found if they are sufficiently distinct and can be resolved in the Fourier-transformed slow time. Second, the association of the determined velocities to the targets is not obvious. In the case of only a few targets per cell and a few paths, a combinatorial search may be feasible. That is, every possible association is attempted and the result with the lowest estimation residual is chosen as the correct velocity estimate.

The final result of this two-step method is a reconstructed image of the scene and corresponding velocity estimates for the detected targets. Note that despite solving K different sparse reconstruction problems, the computational load of the two-step approach is much lower compared to the sparsity-based scheme proposed in [7] that jointly reconstructs the target location-velocity space. Typical sparse reconstruction schemes have at least linear complexity in the number of unknowns [11]. Hence, the complexity of the joint approach scales with the potentially large number of velocity bins N_v . Since we do not need to discretize the target velocities for the two-step approach, the complexity only scales with the number of pulses K . Further, the computational load of the least-squares step is negligible. Hence, we see an overall reduction of complexity by the factor N_v/K .

IV. RESULTS

A. Simulation Results

We present simulation results to demonstrate the effectiveness of the proposed two-step multipath exploitation approach. In all simulation examples, independent and identically distributed complex circular Gaussian receiver noise with a signal-to-noise ratio of 10 dB is added to the measurements before applying the downsampling operation.

Simulations were performed for a wideband pulse-Doppler multistatic radar with one transmitter and uniform linear array with $N = 8$ receivers. The inter-element spacing of the array is 10 cm and the transmitter is assumed in the center of the array. A modulated Gaussian pulse, centered at $f_c = 2$ GHz, with a relative bandwidth of 50% is transmitted. The pulse repetition interval is set to 10 ms and $K = 15$ pulses are processed coherently. At the receiving side, $T = 150$ fast time samples in the relevant interval, covering the target and multipath returns, are taken at a sampling rate of $f_s = 4$ GHz. The front wall is modeled with thickness $d = 20$ cm and relative permittivity $\epsilon = 7.66$, and is located parallel to the array at a distance of 3 m. Two side walls are considered at ± 2 m in crossrange, each of which causes 3 different multipath returns per target depending on whether the secondary reflection at the wall happens on transmit, receive, or both legs of the round trip path. The multipath returns are all considered to

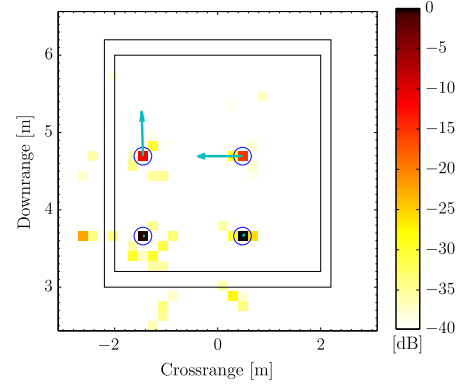


Fig. 1. Two-step reconstruction using 13% of the measurements.

be 6 dB weaker than the direct path. Hence, in total, there are $R = 7$ paths per target contributing to the received signal. We assume that the returns from the front wall have been properly suppressed. We neither consider any wall returns nor any multipath from the back wall located at 6 m downrange. We consider two stationary targets residing at coordinates (0.5, 3.7) m and (-1.5, 3.7) m and two moving targets at (0.5, 4.7) m and (-1.5, 4.7) m, respectively. The moving targets are assumed to be 8 dB weaker than the stationary targets and possess respective velocities $(-0.45, 0)$ m/s and $(0, 0.3)$ m/s. The scene of interest is spatially discretized into an 32×32 pixel grid. Hence, the slowly moving target assumption holds up to approximately 1 m/s.

In Fig. 1, we present the target location reconstruction and velocity estimation using only 13% of the full Nyquist sampled measurements, averaged over 20 Monte Carlo runs. That is, only 20 fast time samples are acquired using a linear mixing scheme by correlating with Gaussian sequences. An image is shown where the magnitudes are accumulated over all paths and pulses. The velocity estimates for the four strongest targets are indicated using arrows. The result of the two-step approach generally exhibits a fairly high background noise compared to the joint reconstruction scheme of [7] (not shown); however, the four targets and the corresponding velocities are well reconstructed.

In order to evaluate the velocity resolution performance of the proposed method as a function of the number of multipath returns, we simulated two targets in the same location but moving with different crossrange velocities. We consider the same paths as described in the first example, but first simulate and exploit only 5, then 6, and finally all 7 paths for velocity estimation. We vary the velocity difference between the two targets from 0.4 m/s to 2 m/s in steps of 0.4 m/s and repeat each simulation 100 times. We assume that the target location is known; hence, only the velocities are estimated from the reflectivities in the multipath components. The target location is chosen such that the observed Doppler velocity is zero for both targets in the direct path, i.e. the targets are in the broadside direction of the array.

The RMSE of the velocity estimates is depicted in Fig. 2a,

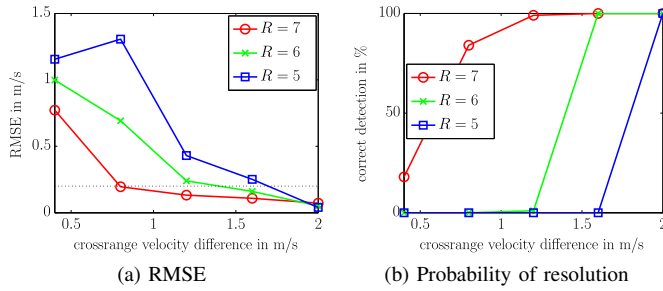


Fig. 2. Velocity resolution performance for various amounts of multipath.

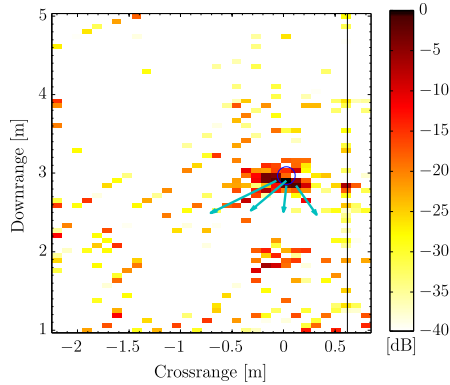


Fig. 3. Two-step reconstruction of the walking human using 20% of the measurements.

whereas the probability of a velocity estimation error lower than 0.2 m/s is shown in Fig. 2b. We observe that velocity estimation performance improves with increasing number of exploited multipath returns. Also, velocity estimation becomes increasingly difficult for decreasing velocity difference between the targets, as evident from the higher RMSE and lower detection results for small velocity differences. This is attributed to the resolution limit of the Doppler processing step.

B. Experimental Results

Experimental results are presented for a wideband real aperture pulse-Doppler radar with a single transmitter and a uniform linear array with $N = 8$ receivers. The data has been recorded at the Radar Imaging Lab, Villanova University, in a semi-controlled lab setup. The transmit waveform is a modulated Gaussian pulse, covering the frequency range of 1.5 to 4.5 GHz. At each receiver, 768 fast time samples have been recorded at a sampling rate $f_s = 7.68$ GHz. Early and late returns have been gated out to clean the data, resulting in $T = 153$ samples. The transmitter was placed 62 cm away from a side wall and the receive array (element spacing 6 cm) was placed on the other side of the transmitter at a distance (to the first element) of 29.2 cm on the same baseline. No front wall was present in the scene. A total of $R = 4$ possible propagation paths are expected, namely, the direct path, two first order and one second order multipath via the side wall. A scenario with a human walking diagonally towards the radar

was recorded. All of the above mentioned propagation paths are expected to be observed for the human.

The proposed two-step method is employed and the imaging result is shown in Figure 3 along with the velocities of the four strongest targets. We apply linear mixing in the fast time to one third of the original samples and use only 5 receivers amounting to 20% of the full measurements. The two-step approach is able to recover the moving human at the correct location and the direction of the estimated velocity vectors is generally consistent with the ground truth. However, the individual velocity estimations of the four scattering centers differ considerably. This may be attributed to the complex nature of human torso and limb movements.

V. CONCLUSION

We have proposed a computationally efficient two-step multipath exploitation method for indoor target localization and estimation of the full target velocity vector. Sparse reconstruction is first employed to localize the targets and also separate the contributions of the different propagation paths, which are then exploited to gain additional velocity information on the targets. We have demonstrated the effectiveness of the approach using simulated and experimental data.

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