Instantaneous frequency and time-frequency signature estimation using compressive sensing

Branka Jokanović*[ab], Moeness Amin[b], Srdjan Stanković[ab]
[a]Elektrotehnički fakultet Podgorica, University of Montenegro, Džordža Vašingtona bb. Podgorica;
[b]Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA

ABSTRACT

This paper considers compressive sensing for time-frequency signal representation (TFSR) of nonstationary radar signals which can be considered as instantaneously narrowband. Under-sampling and random sampling of the signal stem from avoiding aliasing and relaxing Nyquist sampling constraints. Unlike previous work on compressive sensing (CS) and TFSR based on the ambiguity function, reduced observations in the underlying problem are time-domain data. In the reconstruction process, Orthogonal Matching Pursuit (OMP) is used. Since the frequency index in the first iteration of OMP is the same as the one obtained by finding the frequency position of the highest Spectrogram peak, it becomes necessary to consider several OMP iterations to improve over Spectrograms performance. We examine various methods for estimating IF from higher number of OMP iterations, including the S-method. The paper also applies CS for signal time-frequency signature estimations corresponding to human gait radar returns.

Keywords: Instantaneous frequency, micro Doppler, random under-sampling, non-stationary signals, time-frequency distributions

1. INTRODUCTION

Radar is an excellent sensing modality due to its capability of detecting motions of humans. For urban rescue and surveillance operations, including indoor and behind the wall sensing, radars are the modality of choice since they can operate in all types of weather conditions, can penetrate walls and fabrics, and are insensitive to lighting situations inside and outside the enclosed structures [1-4]. It estimates the velocity of a moving object by measuring the frequency shift of the wave radiated or scattered by the object, known as the Doppler effects [5]. For an articulated object such as a walking person, the motion of various components of the body, including arms and legs induces frequency modulations on the returned radar signal and generates sidebands about the Doppler frequency, referred to as micro-Doppler signatures [1, 2]. Gait of a walking person can be observed by continuous-wave (CW), dual frequency, stepped-frequency continuous-wave (SFCW), or pulse-Doppler radar systems [6-11]. Similar to animate targets, inanimate motions, including vibration, oscillations, and rotations produce mono- or multi-component signals, each has a clear instantaneous frequency (IF) law. Characterizing these laws and estimating their respective parameters become important to motion detection and classification [12, 13].

Time-frequency distributions (TFD), in both its nonparametric and parametric forms, are considered a powerful tool for analysis of non-stationary signals and random processes as well as instantaneous frequency (IF) estimation [14-19]. The simplest time-frequency representation is the Short-time Fourier Transform (STFT). The energetic version of this transform is the Spectrogram. Significant improvement of the time-frequency resolution can be achieved using the Wigner distribution (WD). In order to reduce the presence of cross terms in WD, various time-frequency distributions have been proposed. Cohen's class of quadratic time-frequency distributions (QTFD) is a generalization of both Wigner distribution and Spectrograms and can provide high time-frequency resolution with reduced cross terms interference. However, in order to avoid aliasing the distributions from this class require signal oversampling. One quadratic distribution which provides cross-terms free representation without the need for oversampling is S-method [20]. This distribution is based on STFT, which makes it attractive for implementation. Furthermore, the concept of S-method, which is limited convolution, can be applied in obtaining higher order distributions that improve time-frequency representation. Spectrograms can also be used to achieve proper TF distributions though the use of eigen-decomposition schemes [21].

*brankaj@ac.me; phone +382 67 810 860; fax +382 (20) 245 873; www.etf.ac.me

QTFDs, and most existing methods for time-frequency analysis, are defined for uniformly sampled data. Course uniform sampling below the Nyquist rate causes aliasing and must, therefore, be avoided when involving Fourier Transform or Fourier basis. One way to avoid aliasing is by using random sampling scheme. Spectral estimation methods for stationary signals can be applied. Spectral analysis for irregularly sampled signals has been considered in [22]. However, for random sampling of nonstationary signals, these methods become inappropriate for revealing the underlying global and local signal structures, which has also been their shortcomings, even for Nyquistly sampled or oversampled data.

In this paper, we consider compressive sensing for randomly sampled nonstationary signals and for the purpose of IF and microDoppler signature estimation. Compressive sensing has found numerous application in radar [23-26]. Unlike previous work on compressive sensing (CS) and TFSR based on the ambiguity function, reduced observations in the underlying problem are time-domain data [27]. Our approach is based on the fact that non-stationary signals which are instantaneously narrowband are also instantaneously sparse and, as such, can benefit from sparse signal reconstruction methods [24-26]. The local behavior of these signals can be revealed using a time window that slides over the random samples. Two classes of nonstationary signals are considered; one class has its signals uniquely characterized by their IF, the other class is made up of complex signals that exhibit significant changes in their structures over time. The first class includes FM signals. On the other hand, human gait microDoppler signals belong to the second class. It is important to note that from sparse signal representation perspective, signal sparsity in the first class is constant and does not change with window position, whereas sparsity in the second class is time-varying and depends on the window position.

For both nonstationary signal classes discussed above, we perform sparse signal reconstructions over overlapping intervals defined by the different window positions to provide time-frequency signal representations (TFSR). Orthogonal matching pursuit is used as the reconstruction algorithm. Since IF estimate from the first iteration of OMP is the same as that provided by the location of highest peak value in Spectrograms, higher number of iterations becomes necessary if any improvement over Spectrograms is to be achieved. The paper proposes various methods to process the frequencies corresponding to the different OMP iterations, leading to enhanced IF estimation. One of these methods is based on the S-method which showed improvement over Spectrograms under critical and over-sampling conditions. We verify our approaches using synthetic data. We also apply CS to real data corresponding to human gait radar returns. These data are obtained at the Radar Imaging Lab of the Center for Advanced Communications at Villanova University.

The paper is organized as follows. Section 2 covers the theoretical background regarding the time-frequency analysis. Section 3 discusses compressive sensing and OMP as applies to the underlying problem. In Section 4, we cover the issues related to the IF estimation and the need for higher number of OMP iterations. The improvement of instantaneous frequency estimate using S-method is also presented in Section 4. Experimental results are shown in Section 5. Conclusions are given in Section 6.

2. BACKGROUND

2.1 Two Signal Classes

Radar returned signals contain frequency changes which provide information about the target velocity and motions. The shift in the carrier frequency for targets with constant velocity is referred as Doppler frequency. Accelerating, decelerating, vibrating, rotating, oscillating, or maneuvering targets produce time-varying Doppler frequencies. For a mono-component signal, whether the Doppler frequency is fixed or time-varying, the instantaneous frequency information uniquely defines the signal and characterizes its local behavior. Many modern radar systems emits mono-component nonstationary signals such as linear frequency modulated (LFM), also referred to as chirp signals, to achieve pulse compression. Further, FM signals are easily generated and, as such, they are considered the preferred waveforms for smart jammers, and have proven effective in hindering and compromising communications and radar receivers. This class of signal can be represented by,

$$s(t) = Ae^{j(2\pi f_0 t + \phi(t))} \tag{1}$$

where f_0 is the carrier frequency, A is the reflectivity and $\phi(t)$ is the signal phase. The IF is obtained from the derivative of the phase. Joint time-frequency variable representations strive to place the signal power at the IF. As such, frequencies different from the IF would assume zero power. In this respect, the signal local frequency representation has one non-zero occupancy. In compressive sensing terminology, the signal is 1-sparse.

Multicomponent signals, where each component assumes a different IF with fixed or time-varying amplitude, comprise a different class of nonstationary signals where the sparsity level may change from one instant to another, depending on the time-span of each component. A clear example of signals in this class is the Doppler signals associated with Human gait. Whereas the torso produces the same IF for a walking motion, and as such, the same sparsity when considered alone, the limbs, i.e., the arms and legs, exhibit cyclic motions producing replicated, but spaced apart, Doppler signatures. In this case, the combined non-contiguous time-frequency signature imposes different signal sparsity levels over time. This class of signals can be represented by,

$$s(t) = \sum_{k} A_k(t) e^{j\phi_k(t)} \tag{2}$$

For both classes of signals, time or frequency domain analysis cannot provide sufficient information about the nature of the target motion. However, joint time-frequency analysis can be effectively used to analyze radar signals. In this paper, we consider each class separately and show how to estimate the signal IF and complex gait Doppler signature under missing samples or random sampling.

2.2 S-method

The S-method is originally introduced to improve the concentration over the spectrogram, avoiding the cross terms present in the Wigner distribution. It is defined as:

$$SM(t,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\theta) STFT(t, \omega + \frac{\theta}{2}) STFT^*(t, \omega - \frac{\theta}{2}) d\theta, \tag{3}$$

where $P(\Theta)$ is the window function in the frequency domain. If the window size is the same as the auto-term width, the auto-terms concentration as in the Wigner distribution (WD) is achieved, while all cross-terms will be avoided when the distance between signal components is greater than the window length.

In general, the S-method could be considered a new approach to the spectra estimation. In contrast to the well known and widely used smoothed spectrogram which composes two STFTs in the same direction, resulting in the distribution spread, in the S-method two STFTs are composed in a counter-direction, resulting in the concentration improvement. Various forms of the S-method can be defined. For example, order recursive form of the S-method is used for the generalizations and cross-terms free realizations of the L-Wigner, polynomial Wigner-Ville distribution, as well as the Complex-time distribution (CTD) [28, 29]. Note that, in the CTD two spectra with different irregular shapes are convolved, producing high concentration around the instantaneous frequency (IF).

Also, the concept of S-method is used for defining fractional form and affine form of the S-method, composing the windowed form of the fractional Fourier transform and wavelets, respectively. Furthermore, it has been used in synthesis to obtain a signal from a spectrum that could be considered as a sum of the almost Wigner representations of individual signal components. This is similar to the case when the Orthogonal Matching Pursuit (OMP) is used for time-frequency representation. Namely, with the OMP, we can obtain the time-frequency transform that is almost the short-time Fourier transform (STFT), and then by applying the S-method we will obtain OMP based WD that could be considered as almost the WD.

3. COMPRESSIVE SENSING

Let **x** be the discrete–time signal of interest which can be represented as Nx1 vector and suppose that **x** has a sparse representation in a known basis, **Ψ**. That is, the signal can be represented as weighted sum of basis vectors $\psi_1, \psi_2, ..., \psi_N$:

$$x = \sum_{i=1}^{N} s_i \psi_i. \tag{4}$$

The elements s_i form the vector **s** which is a K-sparse vector (i.e., containing exactly K nonzero values, K « N). Suppose that instead of N samples, M linear measurements are acquired. If we denote the measurements as a vector **y**, the following relation holds:

$$\mathbf{y} = \boldsymbol{\phi}\mathbf{x} = \boldsymbol{\phi}\boldsymbol{\psi}\mathbf{s} = \mathbf{A}\mathbf{s}. \tag{5}$$

To reconstruct **s** from **y**, we need to solve an under-determined linear system of equations. Generally, this is ill-posed problem. However, compressive sensing theory asserts that under certain conditions, it is possible to perfectly recover **s** from **y**. The signal reconstruction is performed by using optimization algorithms. There is a number of optimization techniques for finding the sparsest solution of the linear system [30, 31]. They can be divided in two major groups: convex relaxation and iterative greedy search [30, 31]. Convex optimization solves the following optimization problem (l_1-minimization):

$$\hat{s} = \min \|\mathbf{s}\|_{l_1} \textit{ subject to } \mathbf{y} = \mathbf{A}\mathbf{s}. \tag{6}$$

Convex relaxation based methods, though computationally intensive, are important due to their desirable recovery performance. The iterative greedy search methods have lower complexity and hence their use is more attractive in solving large-dimensional CS problems. One of the most commonly used greedy algorithms is Orthogonal Matching Pursuit (OMP), which is applied in this paper. The steps of this iterative procedure are given in **Algorithm 1**. In later sections, we will refer back to some of these steps for comparison. OMP starts with an initial null set. In each iteration, OMP selects a column of **A** that has the highest correlation with the residual of measurements **y**. In the next step, OMP removes the contribution of this column from current residual to compute a new residual. Only one atom is chosen per iteration, followed by adjusting the amplitudes of past atoms which is performed by least-squares procedure. As a stopping criterion, fixed number of iterations or thresholding can be used.

It is important to note that, for the purpose of IF estimation, the first atom in OMP may not end up to be the one with the highest amplitude after more than one iteration are performed. The question as of how to decide on the atom best estimating the IF is considered in the following section.

3.1 Sparse Reconstruction Techniques

Algorithm 1: Orthogonal matching pursuit

Inputs: measurements **y**, matrix **A**, initial residual $\mathbf{r_0}=\mathbf{y}$, $\boldsymbol{\phi}_\mathbf{0}=[\,]$
Output: signal **s**
At each iteration *t* until stopping criterion is met do:

1. Find the column of matrix A which has maximum correlation with current residual.
$$\arg\max_j |< r_{t-1}, A_j >|$$
2. Augment the matrix of chosen atoms.
$$\phi_t = [\phi_{t-1}; A_j]$$
3. Solve the least square problem.
$$s_t = \arg\min \|y - \phi_t s_t\|_2$$
4. Calculate the new residual.
$$r_t = y - \phi_t s_t$$

Problem defined in (6) does not assume the presence of noise. Since, in practice, noise is always present, modifications of previously mentioned algorithms have been proposed. However, they require the knowledge of certain parameters of noise or signal. In the case when the Gaussian noise variance is known, Orthogonal Matching Pursuit can use a threshold as stopping criteria, instead of fixed number of iterations. The threshold which can be used for stopping the procedure is [32]:

$$\|r_i\| \le \varepsilon \qquad \varepsilon = \sigma\sqrt{M + 2\sqrt{M \log M}} \tag{7}$$

This threshold depends on the number of measurements *M* and the standard deviation of the Gaussian noise.

Basis pursuit with inequality constraints

Instead of (6), the following optimization problem is solved:

$$\arg\min \|s\|_1 \text{ s.t. } \|y - As\| \le \varepsilon \tag{8}$$

This minimization can be used for the case when the noise is power limited and that limit is known.

The Dantzig selector

This optimization algorithm is another convex program and can be used for the case of unbounded noise.

$$\arg\min \|s\|_1 \ \text{ s.t. } \ \left\|A^*(y - As)\right\|_\infty \le \varepsilon \tag{9}$$

A* denotes pseudo-inverse matrix.

Since in practice, noise characteristics are usually unknown, the choice of threshold is heuristic. In order to properly estimate the parameter ε, cross validation is suggested [33]. Namely, we divide the measurements into two sets: the estimation set and cross validation set. For each of these sets, there is a corresponding matrix ϕ. The algorithm for the cross validation based Dantzig selector can be described through the following steps:

1. Set $\varepsilon = \alpha \left\|A_E^T \mathrm{y}_E\right\|_\infty$. Over-fitting can be avoided by choosing α=0.99 as suggested [33].
2. Estimate s by using Dantzig selector (9).
3. If $\left\|A_{CV}^T(\mathrm{y}_{CV} - A_{CV}\mathrm{s})\right\|_\infty < \varepsilon$ then set $\varepsilon = \left\|A_{CV}^T(\mathrm{y}_{CV} - A_{CV}\mathrm{s})\right\|_\infty$ and go to step 2. Otherwise, terminate the procedure.

The estimation set is denoted by subscript $_E$, while the cross validation set is denoted by subscript $_{CV}$.

4. HUMAN GAIT CLASSIFICATION AND INSTANTANEOUS FREQUENCY ESTIMATION FOR RANDOMLY SAMPLED DATA

4.1 Problem formulation

Let **x** be a non-stationary signal which, in general, can be wideband, but it is assumed to be narrowband over short time intervals. We consider the time-frequency signal analysis for randomly sampled or significantly under-sampled data. The objective of this paper is to obtain cross terms-free and alias-free time-frequency representation based on the observed incomplete samples. Towards this objective, two remarks are in order:

1. Using incomplete set of samples to compute the signal time-frequency representation introduces noise in the time-frequency domain.

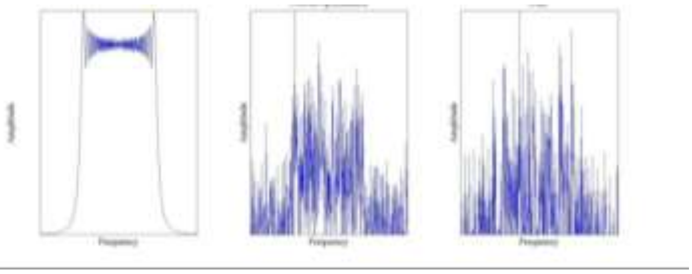
2. The CS approach applied to the entire signal produces poor result due to lack of signal sparsity when considered globally, i.e., over the entire data record. It is noted that most of the non-stationary signals are not sparse in the time or frequency domain. This fact is demonstrated In Figure 1. It is evident that for a chirp signal, neither Convex Optimization nor Greedy algorithm yields appropriate signal reconstruction**.**

4.2 Time-frequency representation of signal with incomplete set of samples

In order to obtain proper time-frequency representation, we propose portioning the data into overlapping segments and carrying signal reconstruction over each segment separately. The problem can be formulated as follows:

$$\min \|\mathbf{LFR}(n_i)\| \ \text{ s.t. } \mathbf{y}_\mathrm{i} = \mathbf{A}_\mathrm{i}\mathbf{LFR}(n_i); \quad \mathbf{y_i} \ne \varnothing; \quad \mathrm{card}\{\mathbf{y_i}\} \ge c\, K \log(N_p / K) \tag{10}$$

where LFR is the signal local frequency representation when viewed from a window, K is the sparsity level within a window, N_p is the window width, c is a constant and y_i is the observation vector. The wide matrix Ai linearly maps the sparse LFR vector to the observation vector. The observations are multiplied by the corresponding coefficients of the window function. Each matrix A_i represents partial Fourier matrix. The rows of A_i are drawn from the rows of the N_p-dimensional discrete Fourier transform (DFT) matrix. The specific rows of the DFT matrix which are used for each window depend on the time indices of the available data within the window. Acquiring the measurements by randomly choosing samples offers certain advantages for OMP. Further, the calculation of the correlation in the OMP first step can be performed using fast Fourier transform. In [34, 35], a comparison is performed for signal reconstruction by using Basis Pursuit and OMP. The results show that the recovery rates for Basis pursuit and OMP are very similar. In some cases, OMP outperforms Basis pursuit [34, 35]. Additionally, running time for Basis pursuit is significantly longer. Since the time-frequency representation is obtained by performing sparse signal reconstruction over highly overlapping short windows covering the entire data record, OMP would be preferred as the reconstruction algorithm.



a) b) c)

Figure 1. a) Fourier transform of full data, b) reconstructed spectrum using convex optimization, c) reconstructed spectrum using Orthogonal Matching Pursuit.

The number of iterations in OMP algorithm

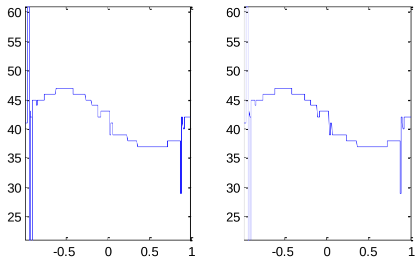
As evident from the OMP algorithm in the previous section, in the first iteration, the algorithm chooses the column A_j that has the largest inner product with the residual. After finding and adding the atom A_j in new dictionary ϕt, a least squared problem is solved in order to obtain new signal estimate.

$$LFR_t(n_i) = \arg\min \left\| y - \phi_t LFR_t(n_i) \right\|_2 \tag{11}$$

For first iteration, A_j is a vector. Therefore, the signal estimate is obtained as:

$$< y, A_j^{-1} > \;\; \Leftrightarrow \;\; < y, \frac{A_j}{\left\| A_j \right\|^2} > \tag{12}$$

Computing the atom, or A_j, with the highest correlation is the same as to choosing the frequency with the highest peak value when computing the Spectrograms (Figure 2.). However, since Spectrogram with missing samples is likely to introduce error in IF estimation, we need to proceed with higher number of iterations when using OMP. This number should be carefully chosen in order to avoid under-fitting or over-fitting of data.



a) b)

Figure 2. Sinusoidal FM signal: a) IF obtained using initial measurements, b) IF obtained by using one iteration in the OMP

Window width

The LFR of the proposed approach has similar properties as the STFT. The time-frequency resolution of the obtained LFR depends on the window width. In order to improve signal representation for STFT in the case of full data, various approaches have been proposed. One approach is based on the use of the quadratic distribution such as the S-method. In the following section, we will adapt this approach to address the IF estimation for nonstationary signals with missing samples. It is well known that in Spectrograms, windows tradeoff temporal and spectral resolutions. However, the window length in the CS case should be sufficiently long to encompass a number of samples that enables sparse signal recovery according to the CS theory. It is important to note that for stationary signals, the window length must conform to Rayleigh criterion. In other words, the mainlobe of the window should be smaller than the frequency separation of the two sinusoids for proper resolution. This is not the case with l_1 signal reconstruction [36], which implies that lower bound on window length can be relaxed from a performance perspective as long as there is sufficient number of samples for reconstruction.

From the above discussion, the window length as well as the number of iterations in the OMP has to be carefully chosen.

4.3 Instantaneous frequency estimation

As mentioned previously, improvement in time-frequency representations for uniformly sampled data can be achieved by using quadratic distributions. For randomly sampled data, we seek to utilize the same known advantages of the S-method over Spectrograms. However, in the underlying problem, the STFTs employed in the S-method are replaced by the outputs of OMP iterations. In essence, the S-method for OMP can be defined similar to the S-method for the STFT:

$$SM(n,k) = |LFR(n,k)|^2 + 2\,\mathrm{Re}\left[\sum_{i=1}^{L} P(i) LFR(n,k+i) LFR^*(n,k-i)\right]. \tag{13}$$

As before, $P(i)$ is a rectangular window whose width in the discrete domain is defined by L. We assume that the sparsity level within a window is unknown, but is not greater than half of the window width, i.e., $N_p/2$. We reconstruct the signal LFR corresponding to different number of iterations in the range,

$$\text{number of iterations } \in [1, \frac{N_p}{2}]\,. \tag{14}$$

Several criteria are examined to determine the appropriate number of iterations in the above range which should be used in conjunction with the S-method.

Bayesian information criterion (BIC)

Since we deal with Fourier basis, our reconstruction process is approximately fitting certain number of sinusoids to the given data. Bayesian information criterion for a number of sinusoidal components M_i can be defined using measurement vector $\mathbf{y}$ of length N [37]:

$$BIC(M_i) = N \ln\{\sum_{n=1}^{N} [y(t_n) - \sum_{i=1}^{M_i} A_i e^{j\omega_i t_n}]^2\} + \rho M_i \ln N \tag{15}$$

The estimated number of sinusoidal components M is a solution to the following optimization problem:

$$M = \underset{M_1, M_{2\ldots}}{\arg\min}\, BIC(M_i) \tag{16}$$

Mean square error (MSE)

Similar to the previous criterion, the estimated number of sinusoids is obtained as:

$$M = \underset{M_1, M_{2\ldots}}{\arg\min}\, MSE(M_i) \quad \text{where} \quad MSE(M_i) = \sum_{n=1}^{N} |\, y(t_n) - \sum_{i=1}^{M_i} A_i e^{j\omega_i t_n}\, |^2\,. \tag{17}$$

Minimum norm difference (MND)

This criterion states that the estimated number of sinusoids is the one with the smallest difference between energy of initial measurements and energy of reconstructed samples on the measurement position, i.e.:

$$M = \underset{M_1, M_{2\ldots}}{\arg\min} \left\| \|y_{M_i}\| - \|r_{M_i}\| \right\| \tag{18}$$

The above three criteria first find the number of iterations which satisfies the condition, and then proceeds to estimate instantaneous frequency by locating the frequency with the highest amplitude. In the following criteria, we try different number of iterations and find the dominant frequency in each. For each number of iteration, M_i , we determine the value V_i and the position P_i (instantaneous frequency) of maximum element within a window reconstructed spectrum. From those two values and over different iterations, we decide on IF through one of the following two methods.

Highest value (HV)

HV method estimates the instantaneous frequency as the frequency with highest value among all iterations:

$$IF = \arg\max_{P_i} V_i \tag{19}$$

Highest occurrence (HO)

If we denote the set of number of occurrences for each P_i as O_i then the IF estimate is obtained as:

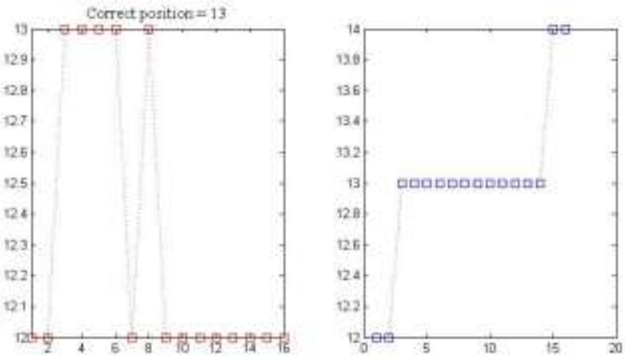
$$IF = \arg\max_{P_i} O_i \tag{20}$$

Our numerical results show that HO method has the highest success rate in IF estimation. From Figure 3, it can be noticed that the instantaneous frequency fluctuates over different iterations. However, in most cases, the correct result has the highest occurrence when S-method is applied. Intuitively, the success of this method can be expected. Namely, Compressive sensing states that there is a probability of exact reconstruction.

The procedure for the proposed IF estimation is described in Algorithm 2.

Algorithm 2: IF estimation using S-method combined with the HO criterion

1. Calculate the local frequency representation (LFR) for different number of iterations.
2. For each LFR compute S-method.
3. Apply HO criterion for each reconstructed window spectrum.
4. Based on the result, estimate the instantaneous frequency.



a) b)

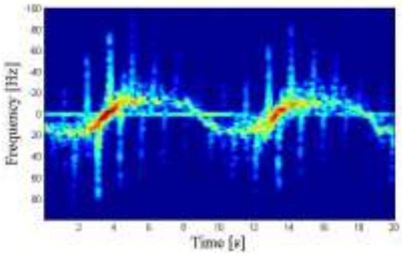
Figure 3. Instantaneous frequency estimation obtained for different number of iterations: a) LFR result, b) S-method based result.

5. SIMULATION RESULTS

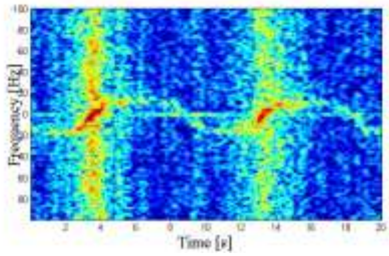
In this section, we demonstrate the effectiveness of the proposed approach using a set of real and synthetic signals The S-method was only applied to IF estimation.

5.1 Signals with time-varying sparsity-Humag Gait

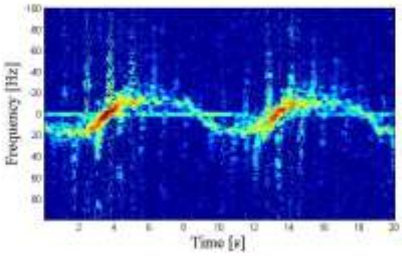
In this experiment, we use the proposed approach for obtaining the time-frequency representation of signals with missing samples. The data represents human gait radar returns. In order to verify the approach, the data is first uniformly sampled at critical Nyquist rate, followed by reduced sampling rate. In one case we use random under-sampling, while in the other we perform uniform subsampling. In both cases 50% of data are missing. The Spectrogram and local frequency representation using OMP both employ Hanning window. The results are shown in Figure 4.The motion signature using Spectrogram on full data depicts detailed torso and limbs movements, as shown in Figure 4. a. Under missing samples, the Spectrogram performance suffers, showing very noisy and cluttered motion time-frequency signature (Figure 4. b). The OMP based result successfully reconstructs the human motion signatures from random samples (Figure 4. c). As it can be seen from the results, randomness plays an important role in avoiding aliasing. If we perform uniform subsampling and reconstruct the spectrum over each window, aliasing would occur (Figure 4. d).



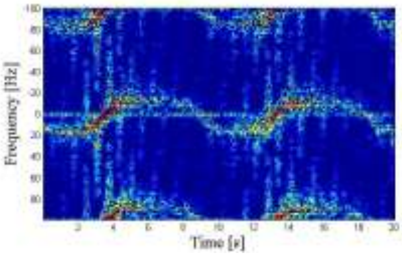
a)



b)



c)



d)

Figure 4. a) Short-time Fourier transform of critically sampled signal, b) Short-time Fourier transform of signal with randomly missing samples (missing samples are replaced by zero value), c) OMP based time-frequency representation from randomly under-sampled signal, d) OMP based time-frequency representation from uniformly under-sampled signal.

5.2 Signals with fixed sparsity- Chirp

In this example, we use our proposed approach and compare instantaneous frequency estimations using different methods and with different number of measurements. The latter is denoted in percentage (first column). For each set of measurements, there are two rows (Table 1). The first row represents the IF estimation from the reconstructed local frequency representation, while the other row denotes the respective results when applying the S-method. For each set, we calculate the percentage of providing correct IF values. The methods used are:

Initial – instantaneous frequency estimation obtained based on the initial measurements,
CO – results obtained by using convex optimization,
BIC – results obtained by using Orthogonal matching pursuit and Bayesian information criterion,
MSE – results obtained by using Orthogonal matching pursuit and Mean square error,
MND – results obtained by using Orthogonal matching pursuit and Minimum norm difference criterion,
HV – results obtained by using Orthogonal matching pursuit and the highest value method,
HO - results obtained by using Orthogonal matching pursuit and the highest occurrence method.

We consider the linear FM signal $x(n) = e^{j\pi(n-128)^2/256}$. Window width is N_p=64.

Table 1. Correct estimation of instantaneous frequency in percentage for different methods of calculation.

	Initial	**CO**	**BIC**	**MSE**	**MND**	**HV**	**HO**
50%	**64**	**79**	**87**	**86**	**73**	**81**	**87**
	77	**86**	**91**	**87**	**74**	**91**	**91**
45%	**63**	**73**	**85**	**83**	**74**	**74**	**86**
	77	**85**	**88**	**88**	**78**	**90**	**91**
40%	**55**	**68**	**85**	**84**	**74**	**65**	**87**
	73	**85**	**90**	**88**	**79**	**89**	**92**

The Table shows that the highest occurrence criterion combined with S-method produces the highest percentage of correct instantaneous frequency values.

6. CONCLUSIONS

In this paper, compressive sensing for IF and time-frequency signature estimation of mono- and multi-component nonstationary signals was applied. The proposed approach is based on the reconstruction of data within short overlapping intervals by using Orthogonal Matching Pursuit. The data over each time-interval has missing samples. We showed that the first iteration in Orthogonal Matching Pursuit is not sufficient for instantaneous frequency estimation. The improvement in IF estimation was obtained by using the S-method applied to the output of OMP over different iterations, followed by a frequency selection based on the notion of highest occurrence. In addition to IF estimation for mono-component signal, the OMP was also applied to human gait data to recover the microDoppler signature under missing samples.

Acknowledgment

This work was supported in part by ONR under grant N00014-11-1-0576 and in part by ARO and ARL under contract W911NF-11-1-0536. The work by Ms. B. Jokanovic and Dr. S. Stankovic was performed during their leave to Villanova University as Visiting Research Associate and Visiting Professor, respectively.

REFERENCES

[1] Skolnik, M., [Introduction to Radar Systems], McGraw Hill, New York, NY (2001).
[2] Clemente, C., Balleri, A., Woodbridge, K. and Soraghan, J., "Developments in target micro-Doppler signatures analysis: radar imaging, ultrasound and through-the-wall radar," EURASIP Journal on Advances in Signal Processing 2013, 2013:47 doi:10.1186/1687-6180-2013-47.
[3] Amin, M. G., [Through-the-wall radar imaging], CRC Press (2010).
[4] Amin, M. G., Ahmad, F., "Wideband synthetic aperture beamforming for through-the-wall imaging," IEEE Signal Processing Magazine, Vol. 25, 110-113 (2008).
[5] Chen, V. C., "Analysis of radar micro-Doppler signature with time-frequency transform," Proc. Of the IEEE Workshop on Statistical Signal and Array Processing (SSAP), 463-466 (2000).
[6] Kim, Y. and Ling, H., "Human Activity Classification Based on Micro-Doppler Signatures Using a Support Vector Machine," IEEE Transactions on Geoscience and Remote Sensing, Vol. 47, No. 5, 1328 - 1337 (2009).
[7] Orovic, I., Stankovic, S. and Amin, M. G., "A new approach for classification of human gait based on time-frequency feature representations," Signal Processing, Vol. 91, Issue 6, 1448–1456 (2011).
[8] Fairchild, D. and Narayanan, R., "Classification and modeling of human activities using empirical mode decomposition with S-band and millimeter-wave micro-Doppler radars," Proc. SPIE, Radar Sensor Technology XVI, Baltimore, MD, May (2012).
[9] Tivive, F. C., Bouzerdoum, A. and Amin, M. G., "A human gait classification method based on radar Doppler spectrograms," EURASIP Journal on Advances in Signal Processing, vol. 2010, Article ID 389716, (2010).
[10] Ahmad, F., Amin, M. G. and Setlur, P., "Through-the-wall target localization using dual-frequency CW radars," in Proceedings of SPIE, vol. 6201, Sensors, and Command, Control, Communications, and Intelligence (C3I) Technologies for Homeland Security and Homeland Defense V, Orlando, FL, (2006).
[11] Tahmoush, D. and Silvious, J., "Stride Rate in Radar Micro-Doppler Images," IEEE Int. Conf. on Systems, Man and Cybernetics, 4323-4328 (2009).
[12] Setlur, P., Amin, M. G. and Thayaparan, T., "Micro- Doppler signal estimation for vibrating and rotating targets," Proceedings of the Eighth International Symposium on Signal Processing and its Applications, Sydney, Australia, (2005)
[13] Setlur, P., Amin, M. G. and Ahmad, F., "Analysis of micro-Doppler signals using linear FM basis decomposition," Proceedings of the SPIE Symposium on Defense and Security, Orlando, FL (2006).
[14] Boashash, B., "Estimating and interpreting the instantaneous frequency of a signal. I. Fundamentals," Proceedings of the IEEE, Vol. 80, No. 4, 520–538, (1992).

[15] Stankovic, LJ. and Stankovic, S., "An analysis of instantaneous frequency representation using time-frequency distributions -Generalized Wigner distribution," IEEE Transactions on Signal Processing, Vol.43, No.2, 549 - 552 (1995).

[16] Amin, M. G. and Williams, W. J., "High spectral resolution time-frequency distribution kernels," IEEE Transactions on Signal Processing, Vol. 46, No. 10, 2796-2804 (1998).

[17] Amin, M. G., "Minimum variance time-frequency distribution kernels for signal in additive noise," IEEE Transactions on Signal Processing, Vol. 44, Issue 9, 2352-2356 (1996)

[18] Amin, M. G., "Time-frequency spectrum analysis and estimation for nonstationary random processes," in Time-Frequency Signal Analysis: Methods and Applications, Editor, B. Boashash, Longman Cheshire (1992).

[19] Jung, A., Tauböck, G. and Hlawatsch, F., "Compressive spectral estimation for nonstationary random processes," Proceedings of the IEEE Int. Conf. on Acoustics, Speech and Signal Processing, 3029-3032 (2009).

[20] Stankovic, LJ., "A method for time-frequency signal analysis," IEEE Transactions on Signal Processing, Vol.42, No.1, 225 - 229 (1994).

[21] Amin, M. G., "Spectral decomposition of the time-frequency distribution kernels," IEEE Transactions on Signal Processing, Vol. 42, Issue 5, 1156-1165, May (1994).

[22] Babu, P., Stoica, P., "Spectral analysis of nonuniformly sampled data – a review," Digital Signal Processing 20, 359–378 (2010).

[23] Candès, E. J., and Wakin, M. B., "An introduction to Compressive Sampling," IEEE Signal Processing Magazine (2008).

[24] Baraniuk, R. and Steeghs, P., "Compressive radar imaging," in Proc. IEEE Radar Conf., Waltham, MA, 128-133 (2007).

[25] Herman, M. and Strohmer, T., "High-resolution radar via compressive sensing," IEEE Transactions on Signal Processing, vol. 57, no. 6, 2275-2284 (2009).

[26] Yoon, Y. S. and Amin, M. G., "Compressive sensing technique for high-resolution radar imaging," Proceedings of the SPIE Symposium on Defense and Security, Orlando, FL, March (2008).

[27] Flandrin, P. and Borgnat, P., "Time-frequency energy distributions meet Compressed Sensing," IEEE Transactions on Signal Processing, Vol. 58, No. 6 (2010).

[28] Stankovic, S., Orovic, I. and Ioana, C., "Effects of Cauchy Integral Formula Discretization on the Precision of IF Estimation: Unified Approach to Complex-lag Distribution and its L-Form," IEEE Signal Processing Letters, Vol. 16, No. 4, 307-310 (2009).

[29] Orovic, I., Orlandic, M., Stankovic, S. and Uskokovic, Z., "A Virtual Instrument for Time-Frequency Analysis of Signals with Highly Non-Stationary Instantaneous Frequency," IEEE Transactions on Instrumentation and Measurements, Vol. 60, No. 3, 791 - 803 (2011)

[30] Donoho, D. L., "Compressed Sensing," IEEE Transactions on Information Theory, Vol. 52, No. 4, 1289 - 1306 (2006).

[31] Tropp, J., and Gilbert, A., "Signal recovery from random measurements via Orthogonal Matching Pursuit," IEEE Transactions on Information Theory, Vol. 53, No. 12, 4655 - 4666 (2007).

[32] Cai, T. T., Wang, L., "Orthogonal Matching Pursuit for sparse signal recovery with noise," IEEE Transactions on Information Theory, Vol. 57, No. 7, 4680 - 4688 (2011).

[33] Gurbuz, A. C., McClellan, J. H., and Scott Jr., W. R., "Compressive sensing for subsurface imaging using ground Penetrating radar," Signal Processing, Elsevier (89) (2009).

[34] Kunis, S., and Rauhut, H., "Random sampling of sparse trigonometric polynomials II - Orthogonal Matching Pursuit versus Basis Pursuit," Foundations of Computational Mathematics, 2008 - Springer (2007).

[35] Rauhut, H., "Stability Results for Random Sampling of Sparse Trigonometric Polynomials," IEEE Transactions on information theory, Vol. 54, No. 12, 5661 - 5670 (2008).

[36] Florescu, A., and Ciochina, S., "Refining accuracy of the spectral lines estimation by a sparsity based approach," Communications (COMM), 2012 9th International Conference on (2012).

[37] Stoica, P. and Selén, Y., "Model-order selection: a review of information criterion rules," IEEE signal processing magazine, Vol. 21, No. 4 (2004).