Source Separation and Localization Using Time-frequency Distributions

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Abstract

In this paper, we describe the role of time-frequency distributions (TFDs) in array processing. We particularly focus on quadratic time-frequency distributions (QTFDs). We demonstrate how these distributions can be properly integrated with the spatial dimension to enhance individual source signal recovery and angular estimation. The framework that enables such integration is referred to as spatial time-frequency distribution (STFD). We present the important milestones in STFDs which have been reached over the last 15 years. Most importantly, we show that array processing creates new perspectives of QTFDs and defines new roles to the auto-terms and cross-terms in both problem formulation and solution. Multi-sensor configurations, in essence, establish a different paradigm and introduces new challenges which did not exist in a single-sensor time-frequency distribution.

Index Terms

Spatial time-frequency distribution, DOA estimation, blind source separation, time-frequency point selection.

I. Introduction and Historical Perspective

Time-frequency signal representations enable separations of nonstationary signals overlapping in both time and frequency domains where windowing and filtering based approaches fail to isolate the different

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signal components [1], [2]. Among numerous nonstationary signals which arise in many passive and sensing modalities, signals with instantaneous frequency (IF) laws, such as frequency modulated (FM) signals, have clear time-frequency (TF) signatures that are contiguous and highly localized. These two properties have led to important advances in nonstationary signal detection and classifications over the past four decades. Both parametric and non-parametric techniques play important roles in characterizing FM signals. Whereas the latter are mainly defined by quadratic time-frequency distributions (QTFDs) which have their roots in Wigner-Ville Distributions [3], the former pursue the estimation of the order as well as the parameters of the FM polynomial phase signal (PPS). Advantages of QTFDs lie in their accommodation of multi-component signals, where each component in the signal can have a different IF. The PPS parameter estimation techniques avoid any bilinear operation and, as such, are not faced with the challenge of eliminating cross-terms which falsely point to signal power concentration regions when using QTFDs. In addition to QTFD and PPS estimation techniques, linear TFDs, such as the short-time Fourier transform (STFT) and wavelet transform, have also been successfully applied to analyze signals with IF characterizations [4].

Similar to the TF signature, the source spatial signature also reveals important information about the source. It enables source discrimination based on the respective angular position as viewed from a receiver array. A source may be an emitter or a reflector of electromagnetic, acoustics, or ultrasound waves. Depending on the propagation environment, the source can be characterized solely by its bearings, i.e., directions-of-arrival (DOA) or through a linear combination of its multipaths. The former characterization is known as steering vectors in which the signal exhibits a phase progression across the different antennas as it traverses the array. In this case, the source spatial signature is characterized by its respective bearing angle that can be provided by DOA estimation techniques. The latter is often referred to as the "generalized" steering vector, and establishes the notion of "mixing". The mixing matrix depends on the corresponding source propagation channel and is a function of unknown array manifold. In this case, we cast the problem as blind source separation (BSS), which can be associated with sensors in either co-located or distributed configurations. The maturities of the two general areas of array processing and TF analysis made the case for developing an integrated approach where the spatial and TF signature estimations interplay to serve both problems. The result is an improved signal localization and separation using time, frequency, and space variables. We tend to refer to this area of research as nonstationary array processing. The importance of this area stems from the fact that nonstationary signals are encountered in various passive and active arrays using different sensing apparatuses.

In this paper, we describe the role of TFD in array processing which has wide applications in radar,

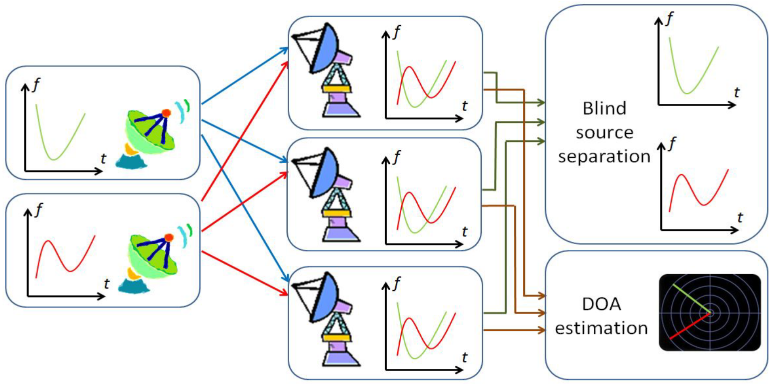


Figure 1. Nonstationary array signal processing.

communications, and satellite navigation. We focus on the class of signals where the IFs uniquely or dominantly define the signal TF signatures. These signals are ubiquitous and can be biological or manmade. Combining the spatial and TF signatures is achieved within a framework referred to as spatial time-frequency distributions (STFDs). This framework utilizes the signal local behavior and power localization for improving both signal-to-noise ratios (SNRs) and source discriminations prior to performing high resolution direction finding and BSS. The STFD requires the computations of the QTFDs of the data received at each antenna, i.e., auto-QTFDs, as well as the cross-QTFDs between each pair of sensors. The STFD framework was originally applied to narrowband signals and then extended to wideband sources.

II. Concept of Spatial Time-Frequency Distributions

Consider an analytic signal vector[1] $\mathbf{z}(t)$ and define the spatial instantaneous autocorrelation function as

$$\mathbf{K}_{\mathbf{zz}}(t,\tau) = \mathbf{z}(t+\frac{\tau}{2})\mathbf{z}^H(t-\frac{\tau}{2}), \tag{1}$$

where $(\cdot)^H$ denotes conjugate transpose (Hermitian) operation. The smoothed spatial instantaneous autocorrelation function is defined as

$$\mathbf{Q}_{\mathbf{zz}}(t,\tau) = G(t,\tau) \underset{t}{*} \mathbf{K}_{\mathbf{zz}}(t,\tau), \tag{2}$$

[1]An analytic signal is a complex valued signal that contains energy in the frequency domain only at positive frequencies. Such a signal is obtained from a real valued signal thanks to the Hilbert transform.

where $G(t,\tau)$ is some time-lag kernel. The time convolution operator $\underset{t}{*}$ is applied to each entry of the matrix $\mathbf{K}_{\mathbf{zz}}(t,\tau)$. The class of quadratic STFDs are then defined as

$$\mathbf{D}_{\mathbf{zz}}(t,f) = \underset{\tau\to f}{\mathcal{F}}\{\mathbf{Q}_{\mathbf{zz}}(t,\tau)\}, \tag{3}$$

where the Fourier transform $\mathcal{F}$ is applied to entry τ of matrix $\mathbf{Q}_{\mathbf{zz}}(t,\tau)$. The discrete time form equivalent to Eq. (3) and Eq. (2) leads to the following implementation of an STFD

$$\mathbf{D}_{\mathbf{zz}}(n,k) = \underset{m\to k}{\mathcal{DF}}\{G(n,m)\underset{n}{*}\mathbf{K}_{\mathbf{zz}}(n,m)\}, \tag{4}$$

which can also be expressed as

$$\mathbf{D}_{\mathbf{zz}}(n,k) = \sum_{m,p=-M}^{M} G(p-n,m)\mathbf{z}(p+m)\mathbf{z}^{H}(p-m)e^{-j4\pi\frac{mk}{N}}, \tag{5}$$

where the discrete Fourier transform $\mathcal{DF}$ and the discrete time convolution operator $\underset{n}{*}$ are applied to entry n of matrix $G(n,m)\underset{n}{*}\mathbf{K}_{\mathbf{zz}}(n,m)$ and matrix $\mathbf{K}_{\mathbf{zz}}(n,m)$, respectively. $N=2M+1$ is the signal length. Note that the diagonal elements of the STFD matrix are termed "*auto-terms*", as they correspond to the quadratic terms associated with each component of the vector $\mathbf{z}(n)$. The off-diagonal elements are termed "*cross-terms*", since they correspond to the bilinear transforms associated with two different components of this vector.

A. STFD properties

Consider a linear model for the vector signal $\mathbf{z}(n)$:

$$\mathbf{z}(n) = \mathbf{A}\mathbf{s}(n), \tag{6}$$

where $\mathbf{A}$ is a $K\times L$ matrix ($K\geq L$) and $\mathbf{s}(n)$ is an $L\times 1$ vector referred to as the source signal vector. Under the above model, the STFDs take the following structure,

$$\mathbf{D}_{\mathbf{zz}}(n,k) = \mathbf{A}\mathbf{D}_{\mathbf{ss}}(n,k)\mathbf{A}^{H}, \tag{7}$$

where $\mathbf{D}_{\mathbf{ss}}(n,k)$ is the source TFD of vector $\mathbf{s}(n)$. Consider an $L\times K$ matrix $\mathbf{W}$, referred to as a whitening matrix such that $\mathbf{WA}$ is a unitary matrix and is denoted as $\mathbf{U}$. That is,

$$(\mathbf{WA})(\mathbf{WA})^{H} = \mathbf{U}\mathbf{U}^{H} = \mathbf{I}, \tag{8}$$

where $\mathbf{I}$ denotes the identity matrix. Pre- and post-multiplying the STFD $\mathbf{D}_{\mathbf{zz}}(n,k)$ by $\mathbf{W}$ leads to the whitened STFD, defined as:

$$\underline{\mathbf{D}}_{\mathbf{zz}}(n,k) = \mathbf{W}\mathbf{D}_{\mathbf{zz}}(n,k)\mathbf{W}^{H} = \mathbf{U}\mathbf{D}_{\mathbf{ss}}(n,k)\mathbf{U}^{H}, \tag{9}$$

where the second equality stems from the definition of $\mathbf{W}$ and Eq. (7). Clearly, the whitening step leads to a linear model with a unitary mixing matrix. Note that the whitening matrix can be computed as an inverse square root of the data covariance matrix [1] or else obtained from the STFD matrices [2] [2]. The STFD structures in Eq. (7) and (9) permit the application of the powerful subspace techniques to solve a large class of problems such as channel estimation, BSS, and high-resolution DOA estimation [1], [5].

B. STFD structure in narrowband array signal processing

When considering L signals arriving at a K-element antenna array, the following linear data model,

$$\mathbf{z}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{n}(n), \tag{10}$$

is commonly assumed, where $\mathbf{z}(n)$ is the $K \times 1$ signal vector received at the array, $\mathbf{s}(n)$ is the $L \times 1$ source signal vector, matrix $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_n]$ represents the propagation matrix, $\mathbf{a}_i$ is the steering vector corresponding to the ith signal, and $\mathbf{n}(n)$ is an additive noise vector whose entries are modeled generally as stationary, temporally and spatially white, zero mean random processes, and independent of the source signals. Under the above assumptions, the expectation of the TFD matrix between the source signal vector and the noise vector vanishes, i.e.,

$$E[\mathbf{D}_{\mathbf{sn}}(n,k)] = 0, \tag{11}$$

and it follows

$$\tilde{\mathbf{D}}_{\mathbf{zz}}(n,k) = \mathbf{A}\tilde{\mathbf{D}}_{\mathbf{ss}}(n,k)\mathbf{A}^H + \sigma^2\mathbf{I}, \tag{12}$$

where $\tilde{\mathbf{D}}_{\mathbf{zz}}(n,k) = E[\mathbf{D}_{\mathbf{zz}}(n,k)]$, $\tilde{\mathbf{D}}_{\mathbf{ss}}(n,k) = E[\mathbf{D}_{\mathbf{ss}}(n,k)]$, and σ^2 denotes the noise power. Under the same assumptions, the data covariance matrix, commonly used in array signal processing, has the following structure

$$\mathbf{R}_{\mathbf{zz}} = \mathbf{A}\mathbf{R}_{\mathbf{ss}}\mathbf{A}^H + \sigma^2\mathbf{I}, \tag{13}$$

where $\mathbf{R}_{\mathbf{zz}} = E[\mathbf{z}(n)\mathbf{z}^H(n)]$ and $\mathbf{R}_{\mathbf{ss}} = E[\mathbf{s}(n)\mathbf{s}^H(n)]$. From relations (12) and (13), it is clear that the STFD and the covariance matrices exhibit the same eigen-structure. This structure is often exploited to estimate signal parameters through subspace-based techniques [6], [7].

[2]Note that while the computation of the whitening matrix from the covariance matrix assumes independent source signals, its computation from the STFD matrices does not require such assumption.

C. Advantages of STFDs over Covariance matrix

STFD-based methods can handle signals corrupted by interference occupying the same frequency band and/or the same time segment but with different TF signatures, improving signal selectivity over approaches using the covariance matrix. In addition, the effect of spreading the noise power while localizing the source signal power in the TF plane increases the effective SNR and provides robustness with respect to noise. Quantitative evaluation of such improvement can be found in [2]. If one selects the kernel $G(n,m)$ in Eq. (4) so that the corresponding TFD satisfies the marginal condition ([3], Section 6.1), then we obtain

$$\sum_{k} \tilde{\mathbf{D}}_{\mathbf{zz}}(n,k) = E[\mathbf{z}(n)\mathbf{z}^H(n)] = \mathbf{R}_{\mathbf{zz}}. \tag{14}$$

Therefore, $\mathbf{R}_{\mathbf{zz}}$ is a low-dimension representations of $\tilde{\mathbf{D}}_{\mathbf{zz}}(n,k)$. In fact, this is the reason that the STFD-based methods offer better performance, such as signal selectivity, interference suppression and high resolution, than conventional covariance matrix based approaches.

D. Other Spatial Time Frequency Representations

A similar STFD framework can be provided using linear transforms, such as the STFT and wavelet transform, leading to the following spatial STFT

$$\mathbf{S}_{\mathbf{z}}(n,k) = \underset{m\to k}{\mathcal{DF}}\{h(m-n)\mathbf{z}(m)\}, \tag{15}$$

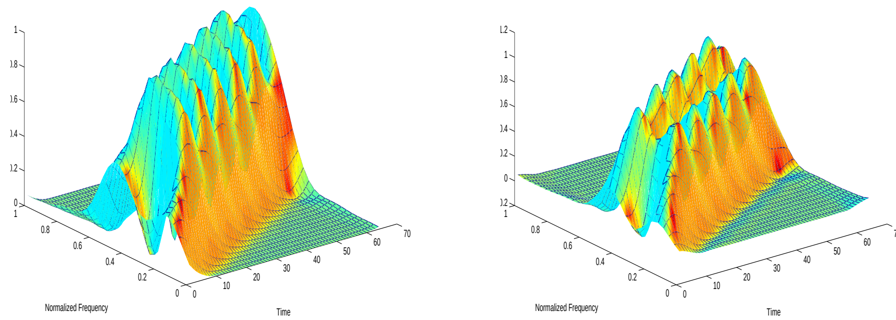
where $h(n)$ is a windowing function. Under the linear model (6), the spatial STFT retains the same structure but with higher dimensionality:

$$\mathbf{S}_{\mathbf{z}}(n,k) = \mathbf{A}\mathbf{S}_{\mathbf{s}}(n,k). \tag{16}$$

These transforms trade off temporal and spectral resolutions, and their squared magnitudes are already considered within the STFD framework. Moreover, multi-resolution analyzes are not most effective for signals characterized by their IF laws. Fig. 2(a) illustrates the discrimination problem between two closely spaced chirp signals when dealing with the STFT. In contrast, in Fig. 2(b), the use of a QTFD allows the two signals to be resolved. Therefore, the STFD, incorporating QTFD, enables source separation and thus estimation of the respective DOAs. In practice, TF discrimination is generally performed through TF point selection procedures, as discussed in the next section.

III. Time-frequency Point Selection

The advantages of TF-based BSS and DOA estimation can only be materialized if appropriate TF points are selected in the formulation of the STFD matrices.



(a) (b)

Figure 2. (a) Square modulus of the STFT of two closely spaced chirps. (b) QTFD of two closely spaced chirps as used in a STFD framework.

A. Time-Frequency point properties

The STFD framework assumes the TF signatures of the source signals are sufficiently different to satisfy one or two of the following conditions:

(a) There exist TF points (n_l, k_l) which correspond to individual source auto-terms. In other words, if $D_{s_is_j}(n,k) = (\mathbf{D_{ss}}(n,k))_{ij}$, then

$$D_{s_is_j}(n_l, k_l) = \delta_{i,j} D_{i,j,l}, \tag{17}$$

and, for each of the L sources i, there is at least an l-th TF point, such that $D_{i,i,l} \neq 0$. $\delta_{i,j}$ is the Kronecker delta, i.e., $\delta_{i,j} = 0$ if $i \neq j$ and 1 otherwise. $D_{i,j,l}$ ($D_{i,i,l}$) is the value of the QTFD between the sources s_i and s_j (or s_i) at the TF point (n_l, k_l).

(b) There exist TF points (n_l, k_l) which correspond to cross-terms. That is,

$$D_{s_is_j}(n_l, k_l) = (1 - \delta_{i,j}) D_{i,j,l}. \tag{18}$$

The above two assumptions imply that, in the STFD framework, the source TF signatures should not be strongly overlapping. The "sufficiently" different signatures represent the "known" discriminating property about the sources required for applications of blind methods.

B. Time-frequency point categorization

Figure 3 shows the real part of the source STFD matrix

$$\mathbf{D_{ss}}(n,k) = \begin{pmatrix} D_{s_1s_1}(n,k) & D_{s_1s_2}(n,k) \\ D_{s_2s_1}(n,k) & D_{s_2s_2}(n,k) \end{pmatrix}$$

when the Pseudo Wigner-Ville distribution (PWVD) is used with a Hamming Window of size 65. In this example, the two signals of interest, $s_1(n)$ and $s_2(n)$, are linear FM (LFM, or chirp) signals. They cross at TF point (128,0.3). The time scale is $[0, 255]$ and the frequency scale is given in normalized frequency (limited to $[0, 0.5]$ which corresponds to the first 128 frequencies obtained by discrete Fourier transform divided by 256). As observed in Fig 3, four types of TF points are observed:

- Type I: TF points that correspond to source auto-terms only. For those points, the source TFD matrix is **a rank-one diagonal matrix**[3].
- Type II: TF points that correspond to source cross-terms only. For those points, the source TFD matrix is **off-diagonal**[4].
- Type III: TF points that correspond to both source cross- and auto-terms. For those points, the source TFD matrix does not exhibit an algebraic structure that could be directly exploited.
- Type IV: TF points where there are neither source cross-terms nor source auto-terms.

The diagonal and off-diagonal structures of types I and II are generally distorted when the sources are mixed. Only the types I and II are of interest to the DOA estimation and BSS problems. The others should be discarded since they do not play any role in either problem. Type IV points can be removed by applying appropriate thresholding. It is difficult to do the same for type III. When using a QTFD with reduced interferences (such as spectrogram, smoothed pseudo Wigner-Ville distribution (SPWVD) [3], or based on particular kernels like in [8]), Type II points will assume small or zero values. However, Type III points will persist, in particular, when the sources have overlapping signatures. With reduced interference distribution, it becomes easier to select Type I points, but often at the expense of auto-terms localization (see Fig. 2). Disjoint sources eliminate a great majority of type III points and facilitate the automatic selection of type I and II points. Such property has been exploited in linear TF-based approaches (principle of the degenerate unmixing estimation technique (DUET) algorithm [9]). Sophisticated time-frequency point selection procedures are generally required, as discussed in the next section. Despite the relative effectiveness of these procedures, cross-terms remain undesirable and discouraged when they extensively clutter the TF domain.

[3]The rank-one diagonal matrix is the "only" possibility of diagonal matrix due to the "middle-point" rule which defines the cross-terms geometry [3].

[4]A matrix is said to be off-diagonal if its diagonal entries are zeros.

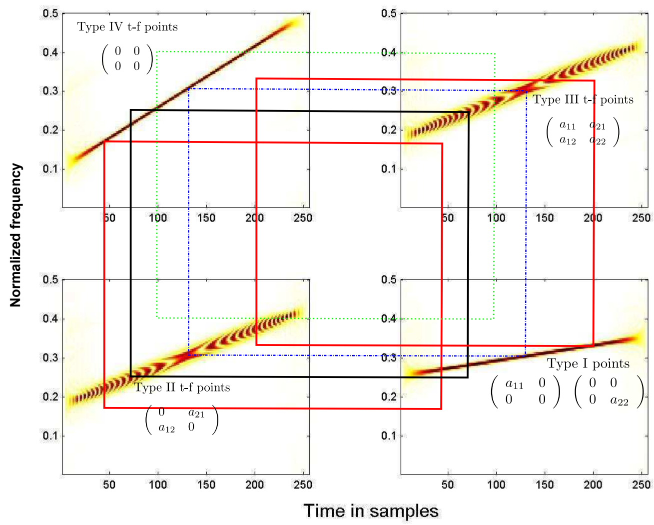


Figure 3. Real part of $\mathbf{D_{ss}}(n,k)$ when the PWVD is used for two LFM signals. TF point types: source auto-terms only (red squares), source cross-terms only (black square), both source cross-terms and auto-terms (blue square) and neither source cross-terms nor source auto-terms (green square).

C. Time-Frequency point selection procedures

Due to (7) and (9) and the fact that in type I (resp. II) points $\mathbf{D_{ss}}(n,k)$ have a very particular algebraic structure (rank-one diagonal matrix resp. off-diagonal matrix), a natural way to tackle the BSS or DOA problems will be to use matrix decomposition algorithms which happens to be a rather classical approach in BSS. Yet, the problem of the automatic TF points selection, in the general case, is not simple. Several TF point selection procedures have been suggested[5], some of which operate in a whitened context, while others do not require such preprocessing. In a whitened context, some procedures utilize matrix trace invariance under unitary transform, making it possible to decide on the presence of source auto-terms. One procedure states that [11]:

For type II points, select STFD matrices that verify:

$$\frac{\operatorname{trace}\{\underline{\mathbf{D}}_{\mathbf{zz}}(n,k)\}}{\|\underline{\mathbf{D}}_{\mathbf{zz}}(n,k)\|} < \varepsilon, \tag{19}$$

where $\operatorname{trace}\{\cdot\}$ denotes matrix trace, $\|\cdot\|$ Frobenius norm and ε is a small user-defined positive scalar

[5]In [10], an exhaustive panorama of all existing detectors is provided and these detectors are compared on synthetic signals involving multi-components correlated sources.

(in section V, these matrices will be denoted by $(.)^c$). For type I points, select STFD matrices that verify:

$$\frac{\mathrm{trace}\{\underline{\mathbf{D}}_{\mathbf{zz}}(n,k)\}}{\|\underline{\mathbf{D}}_{\mathbf{zz}}(n,k)\|} > \varepsilon. \tag{20}$$

(in section V, these matrices will be denoted by $(.)^a$). There is a potential problem with the above detector concerning the selection of type I points. If a non-negligible value of the trace qualifies the presence of source auto-terms, it does not necessarily mean the absence of source cross-terms. In essence some of the selected TF points could be type III points. This problem is addressed in [10] where, under the assumption that the STFD exhibits a Hermitian symmetry, the real value property of the auto-terms and the complex value property of the cross-terms are exploited. A follow-on work accounted for the fact that cross-terms can take real values. Another approach, for type I points, identifies rank-one matrices [12]. More recently, a detector based on the use of the Hough transform has been suggested [8].

In a non-whitened context, most procedures take advantage of the fact that the source TFD is a diagonal rank one matrix at an auto-term TF point of a single source. One way to check whether a matrix is rank one is to use a singular value decomposition (SVD). For example, it is stated in [13] that, for type I points, select STFD matrices that verify:

$$\begin{cases} \mathcal{C}(n,k) = \frac{\lambda_1(n,k)}{\sum_{k=1}^{K}\lambda_k(n,k)} > 1-\varepsilon, \\ \sum_{i=1}^{K}\lambda_i(n,k) > \varepsilon', \end{cases} \tag{21}$$

where $\lambda_i(n,k), i = 1, \cdots, K$, are the singular values of the STFD matrix $\mathbf{D}_{\mathbf{zz}}(n,k)$. The latter are sorted in a decreasing order. The parameters ε and ε' are some small positive user-defined scalars. Another procedure was proposed in [14] for "quasi-disjoint" sources. In [10], a slight modification was suggested to cast off the "quasi-disjoint" assumption. In a noisy environment, the selection of TF points of peak power (type I and type II TF points) may become challenging when the signals are highly corrupted by noise. The spatial diversity, embedded in the STFD matrix, can reduce noise and enhance the TF signatures of the signals of interest. This is achieved by averaging the TFDs over all receiver sensors [15], [16]. In [17], [18], noise is considered within the Neyman-Pearson framework. The best TF point selection method would depend on the data, but focusing on type I points by searching of rank-one matrices is likely to give desired performance.

IV. Time-Frequency DOA Estimation

The advantages of using STFD for DOA estimation stem from the fact that TF point selection, as discussed previously, permits DOA estimation to be performed using STFDs for one or few signal arrivals with specific TF signatures [2]. This means that direction finding can be performed with the number of array sensors smaller than the number of impinging signals. In essence, separate STFDs can be constructed, each corresponding to one source, to perform individual DOA estimation. This would require, however, DOA estimation to be repeated for each signal. By selecting TF points with high signal power, the overall SNR can be improved. This improvement over standard covariance matrix counterparts is more pronounced when the input SNR is low. The offerings of the STFD-based approach extend to all DOA estimations based on second-order statistics, and have been utilized to develop TF-based methods underlying, e.g., MUSIC and maximum likelihood (ML) [6], [5]. The spatial quadratic distribution can be extended to joint-variable domain distributions, such as the spatial ambiguity function (SAF) [19]. The SAF have different features from STFD, e.g., the signal auto-terms are positioned at and around the origin, making it easier to avoid cross-terms in matrix constructions. While STFD-based DOA estimation was first developed and examined using the narrowband signal model, it was thereafter considered for wideband signal platforms [20], [21]. For signals whose IFs can be modeled and finitely parameterized, proper linear transformations provide effective alternatives to STFD-based techniques. For example, FM signals can be made stationary so that the resulting sinusoids can be effectively processed by filtering to achieve source discrimination and noise mitigation prior to performing DOA estimation [22].

A. Time-Frequency MUSIC

To describe conventional MUSIC, we denote $\hat{\mathbf{R}}_{\mathbf{zz}}$ as the estimated covariance matrix of data vector $\mathbf{z}(n)$, and $\hat{\mathbf{G}}$ as the noise subspace of $\hat{\mathbf{R}}_{\mathbf{zz}}$. We use $\hat{\cdot}$ to emphasize that the results are estimated. The MUSIC technique estimates the DOAs by determining the L values of θ for which the following spatial spectrum is maximized,

$$f_{\mathrm{MU}}(\theta) = \left[\mathbf{a}^H(\theta)\hat{\mathbf{G}}\hat{\mathbf{G}}^H\mathbf{a}(\theta)\right]^{-1}, \tag{22}$$

where $\mathbf{a}(\theta)$ is the steering vector corresponding to θ. Similarly, for TF-MUSIC which selects TF regions belonging to L_o signals ($L_0 \leq L$), we denote $\mathbf{G}^{\mathrm{tf}}$ as the noise subspace of the STFD matrix $\hat{\mathbf{D}}_{\mathbf{zz}}$. The noise subspace $\hat{\mathbf{G}}^{\mathrm{tf}}$ can be obtained based on multiple selected TF points and by using either joint block-diagonalization (JBD) [6] or TF averaging [2]. For Q selected TF points $\mathbf{D}_{\mathbf{zz}}(n_l, k_l), l = 1, ..., Q$, the joint block-diagonalization provides $\mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_K]$ as follows:

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \sum_{l=1}^{Q} \sum_{i,p=1}^{K} \left|\mathbf{u}_i^H \mathbf{D}_{\mathbf{zz}}(n_l, k_l)\mathbf{u}_p\right|^2. \tag{23}$$

Matrix $\hat{\mathbf{U}}$ is then partitioned into the estimated signal and noise subspaces. The TF averaging, on the other hand, is a much simpler alternative which provides the eigen-matrix of $\mathbf{D}_{\mathbf{zz}}(n_l, k_l), l = 1, ..., Q$, through eigen-decomposition of $\hat{\mathbf{D}} = \sum_{l=1}^{Q} \mathbf{D}_{\mathbf{zz}}(n_l, k_l)$. Once the noise subspace is obtained, the DOAs are determined by locating the L_0 peaks of the spatial spectrum,

$$f_{\mathrm{MU}}^{\mathrm{tf}}(\theta) = \left[\mathbf{a}^H(\theta)\hat{\mathbf{G}}^{\mathrm{tf}}\left(\hat{\mathbf{G}}^{\mathrm{tf}}\right)^H \mathbf{a}(\theta)\right]^{-1}. \tag{24}$$

In [2], the variance of the estimated DOA is analytically examined using LFM signals as examples. Below, we demonstrate the advantages of TF-MUSIC, as compared to conventional MUSIC. The IF laws of the LFM signals are assumed to be perfectly known.

Example [2]. Consider a uniform linear array of 8 sensors with an inter-element spacing of half a wavelength, and an observation period of 1024 samples. Two LFM signals are emitted from two sources positioned at angles θ_1 and θ_2. The start and end frequencies of the signal source at θ_1 are $f_{s1} = 0$ and $f_{e1} = 0.5$, whereas the corresponding two frequencies for the other source at θ_2 are $f_{s2} = 0.5$ and $f_{e2} = 0$, respectively. PWVD with rectangular window of size $H = 129$ is used to compute the TFD, and TF averaging is used to compute the noise subspace. Fig. 4 displays the root-mean-square error (RMSE) of the estimated DOA $\hat{\theta}_1$ versus SNR for conventional MUSIC, TF-MUSIC, and the Cramer-Rao lower bound (CRLB), where $(\theta_1, \theta_2) = (-10^\circ, 10^\circ)$. Both signals were selected when performing TF-MUSIC ($L_0 = L = 2$). The results were averaged over 100 independent Monte-Carlo runs. The advantages of TF-MUSIC in low SNR cases are evident from this figure. The deviation of the simulation results from the theoretical results for low SNR is because only the lowest coefficient order of the perturbation expansion is used in deriving the theoretical results [2]. Fig. 5 shows examples of the estimated spatial spectrum

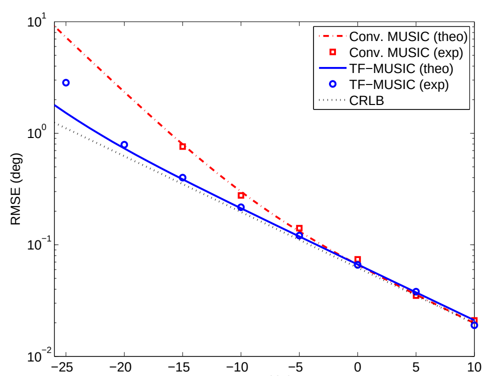


Figure 4. RMSE of DOA estimation versus input SNR.

based on TF-MUSIC and the conventional MUSIC where the angle separation is small ($\theta_1 = -2.5^\circ$, $\theta_2 = 2.5^\circ$). The input SNR is -5 dB. The TF-MUSIC algorithm is performed separately for two sets of

TF points, each belonging to one source (i.e., $L_0 = 1$ in each TF-MUSIC operation). It is evident that the two signals are resolved by the TF-MUSIC whereas the conventional MUSIC fails.

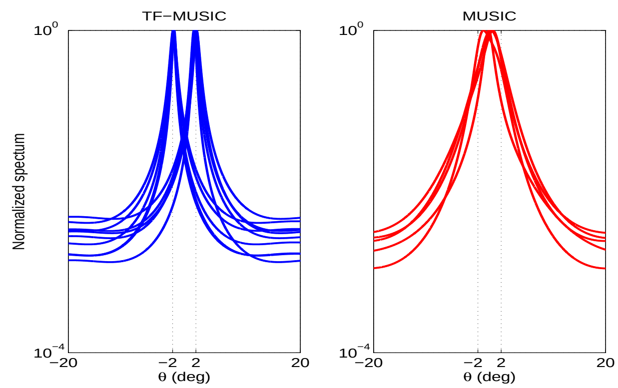


Figure 5. Spatial spectra of MUSIC and TF-MUSIC for closely spaced signals.

B. *Time-Frequency Maximum Likelihood Method*

Consider array observations $\mathbf{z}(1), \mathbf{z}(2), \ldots, \mathbf{z}(N)$, as described in (10) where the mixing matrix $\mathbf{A}(\theta)$ is represented as a function of the DOAs θ. For conventional ML methods, the log-likelihood function, after omitting the constant terms, is given by

$$\mathcal{L}(\theta) = -\frac{1}{\sigma_n^2}\sum_{i=1}^{N}\left[\mathbf{z}(n) - \mathbf{A}(\theta)\mathbf{s}(n)\right]^H \left[\mathbf{z}(n) - \mathbf{A}(\theta)\mathbf{s}(n)\right]. \tag{25}$$

The ML estimate of θ is obtained as the following minimizer:

$$\hat{\theta} = \arg\min_{\theta} \operatorname{trace}\left\{\left[\mathbf{I} - \mathbf{A}(\theta)(\mathbf{A}^H(\theta)\mathbf{A}(\theta))^{-1}\mathbf{A}^H(\theta)\right]\hat{\mathbf{R}}_{\mathbf{zz}}\right\}. \tag{26}$$

We now consider the TF-ML method. We select $L_0 \le L$ signals in the TF domain. The TF-ML estimate of θ_0, i.e., the DOAs of the selected L_0 signal arrivals, is obtained as the following minimizer, which replaces $\hat{\mathbf{R}}_{\mathbf{zz}}$ in (26) by $\hat{\mathbf{D}}_{\mathbf{zz}}$ [5],

$$\begin{aligned}\hat{\theta}_0 = \; & \arg\min_{\theta_0} \operatorname{trace}\left\{\left[\mathbf{I} - \mathbf{A}(\theta_\mathbf{0})\left(\mathbf{A}^H(\theta_\mathbf{0})\mathbf{A}(\theta_\mathbf{0})\right)^{-1}\right.\right. \\ & \left.\left. \cdot\mathbf{A}^H(\theta_\mathbf{0})\right]\hat{\mathbf{D}}_{\mathbf{zz}}\right\}.\end{aligned} \tag{27}$$

Similar to TF-MUSIC, signal localization in the TF domain enables us to select fewer signal arrivals and thus improves the DOA estimates, particularly when the signals are closely spaced. In addition, because ML performs multi-dimensional search, selection of fewer sources also reduces the dimension of the search space for significant reduction of the computational complexity.

In [5], an example is given to show that the TF-ML can estimate the respective DOAs of two closely spaced coherent sources (identical sinusoidal frequencies with a constant phase difference), whereas TF-MUSIC fail to separate these two coherent sources.

C. DOA Estimation of Wideband Nonstationary Signals

DOA estimation for wideband signals, for which the steering vector is frequency-dependent, is different from the narrowband signal counterpart. Conventionally, wideband signals are decomposed into a set of narrowband signal components using the Fourier transform. The resulting narrowband signals can then be processed either incoherently or coherently. Coherent processing techniques align the phases of the narrowband signals before they are combined. An effective and commonly used technique is the coherent signal-subspace processing, which uses a set of focusing matrices to map the steering vector at each frequency into that at a reference frequency prior to coherent combining. The incoherent processing techniques, on the other hand, avoid phase alignment. For example, the output power of narrowband Capon beamformers can be combined using the arithmetic or geometric averaging operation. Generally, the coherent techniques are often preferred due to the superior performance compared to the incoherent counterparts.

TF analysis provides a convenient platform to apply coherent signal-subspace for LFM signals and other nonstationary signals with clear IF laws. Because LFM signals are instantaneous narrowband, focusing matrices can be easily applied to TF points, and the decomposition of the LFM signals into a spectrum of frequency bins is inherently performed in the TF analysis. By assuming that the wideband signals are separable in the TF domain and their IFs do not rapidly change, [20] uses a sufficiently short sliding window to construct the STFD matrices so as to preserve the narrowband structure of the array manifold. The focusing matrices are then applied to the STFD matrices at selected TF points corresponding to the source TF signatures. In [21], the wideband DOA estimation problem is considered in the ambiguity domain for LFM signals with known chirp rates. This method utilizes the fact that the auto-ambiguity function of an LFM signal yields a phase progression that is proportional to the chirp rate and the DOA-dependent delay but is independent of the initial delay and frequency of the LFM signal. As such, the DOAs can be obtained because the LFM parameters are known *a priori*.

V. TIME-FREQUENCY SOURCE SEPARATION

A BSS problem consists of recovering the original waveforms of the source signals without any knowledge of their linear mixture. Two types of inherent indeterminacy exist in a BSS problem, i.e., source signals can only be identified up to a fixed permutation and some complex factors [23]. When the spectral content of the source signals is time-varying, one can exploit the powerful tool of the STFDs to separate and recover the incoming signals. In this context, the BSS problem can be regarded as signal synthesis from the TF plane with the incorporation of the spatial diversity provided by the antennas. In contrast to conventional BSS approaches, the STFDs-based signal separation techniques allow separation of correlated Gaussian sources with identical spectral shape, provided that the sources have different TF signatures.

Herein, the multi-antenna signal $\mathbf{z}(n)$ is assumed to be nonstationary and to obey the linear model (6). The problem under consideration consists of identifying the matrix $\mathbf{A}$ and/or recover the source signals $\mathbf{s}(n)$ up to a fixed permutation and some complex factors. By selecting auto-term points, the whitened auto STFDs have the following structure

$$\underline{\mathbf{D}}_{\mathbf{zz}}^{a}(n,k) = \mathbf{U}\mathbf{D}_{\mathbf{ss}}^{a}(n,k)\mathbf{U}^{H} \tag{28}$$

with $\mathbf{D}_{\mathbf{ss}}^{a}(n,k)$ denoting a diagonal matrix. The missing unitary matrix $\mathbf{U}$ can be retrieved up to permutation and phase shifts by joint diagonalization (JD) of a combined set $\{\underline{\mathbf{D}}_{\mathbf{zz}}^{a}(n_l,k_l)|l=1,\cdots,P\}$ of P auto STFDs. The incorporation of several auto-term points in the JD reduces the likelihood of having degenerate eigenvalues and increases robustness to a possible additive noise. The above JD is defined as the maximization of the following criterion:

$$C_{\mathrm{JD}}(\mathbf{V}) \stackrel{\mathrm{def}}{=} \sum_{l=1}^{P}\sum_{i=1}^{L} |\mathbf{v}_i^H \underline{\mathbf{D}}_{\mathbf{zz}}^{a}(n_l,k_l)\mathbf{v}_i|^2 \tag{29}$$

over the set of unitary matrices $\mathbf{V} = [\mathbf{v}_1,\ldots,\mathbf{v}_L]$. The selection of cross-term points leads to the whitened cross STFD,

$$\underline{\mathbf{D}}_{\mathbf{zz}}^{c}(n,k) = \mathbf{U}\mathbf{D}_{\mathbf{ss}}^{c}(n,k)\mathbf{U}^{H} \tag{30}$$

with $\mathbf{D}_{\mathbf{ss}}^{c}(n,k)$ an off-diagonal matrix. The unitary matrix $\mathbf{U}$ is found up to permutation and phase shifts by joint off-diagonalization (JOD) of a combined set of Q cross STFDs, $\{\underline{\mathbf{D}}_{\mathbf{zz}}^{c}(n_l,k_l)|l=1,\cdots,Q\}$. The JOD is defined as the maximization of the following criterion:

$$C_{\mathrm{JOD}}(\mathbf{V}) \stackrel{\mathrm{def}}{=} -\sum_{l=1}^{Q}\sum_{i=1}^{L} |\mathbf{v}_i^H \underline{\mathbf{D}}_{\mathbf{zz}}^{c}(n_l,k_l)\mathbf{v}_i|^2 \tag{31}$$

over the set of unitary matrices $\mathbf{V} = [\mathbf{v}_1,\ldots,\mathbf{v}_L]$. The unitary matrix $\mathbf{U}$ can also be found up to permutation and phase shifts by a combined JD/JOD of the two sets $\{\underline{\mathbf{D}}_{\mathbf{zz}}^{a}(n_l,k_l)|l=1,\cdots,P\}$ and $\{\underline{\mathbf{D}}_{\mathbf{zz}}^{c}(n_l,k_l)|l=1,\cdots,Q\}$. Note that with the introduction of the STFD framework and with the goal of source separation, cross-terms in the TF plane are no longer undesirable components, as in the case of single sensor processing, indicating false manifestation of energy. Rather, cross-terms and auto-terms assume equal roles in multi-sensor signal processing. Once the unitary matrix $\mathbf{U}$ is determined from either the JD, the JOD or the combined JD/JOD, an estimate of the mixing matrix $\mathbf{A}$ can be computed by the product $\mathbf{W}^{\#}\mathbf{U}$, where $\mathbf{W}$ is the whitening matrix and $(\cdot)^{\#}$ denotes the pseudo-inverse operator. An estimate of the source signals $\mathbf{s}(n)$ can then be obtained from the product $\mathbf{A}^{\#}\mathbf{z}(n)$. BSS can also be performed by exploiting directly the auto or cross STFDs (7) without relying on the whitening step [24][13][10]. Note that the latter usually establishes a bound on the reachable performance.

One of the most important contributions of the STFD framework to BSS problems is enabling solutions of the under-determined problem where there are more sources than sensors (i.e., $L > K$). Herein, for the resolution of the under-determined problem, we review a STFD-based BSS method [14]. We start by selecting auto-term points where only one source exists. The corresponding STFD has then the following form,

$$\mathbf{D}_{\mathbf{zz}}(n,k) = D_{s_i s_i}(n,k)\mathbf{a}_i\mathbf{a}_i^H, \quad (n,k) \in \Omega_i, \tag{32}$$

where Ω_i denotes the TF support of the ith source. The main idea of this algorithm is to cluster together the auto-term points associated with the same principal eigenvector of $\mathbf{D}_{\mathbf{zz}}(n,k)$ representing a particular source signal. Once the clustering and classification of the auto-terms is performed, the estimates of the source signals are obtained from the selected auto-terms using a TF synthesis algorithm [25]. Note that the missing auto-terms in the classification, often due to intersection points, are automatically interpolated in the synthesis process. An advanced clustering technique of the above auto-terms based on Gap statistics is proposed in [26]. Note that a byproduct of the above clustering procedures in the STFD framework is the estimation of the source number.

VI. Applications

Array processing for source separation and localization is important in a wide variety of applications. Conventional array signal processing algorithms assume stationary signals and mainly call for the covariance matrix estimation of the data vectors. In contrast, TF-based array processing techniques, as discussed above, exploit signal nonstationarity and, as a result, offer improved source separation and localization capabilities for use in many applications, such as wireless communications, navigation, radar, sonar, underwater communications, and biomedical systems. Some application examples are discussed below.

In wireless communications, FM signal representations can be used for pulse shaping in single- or multi-user communications. In the latter, multiple sets of FM waveforms are designed to have distinct IF laws so as to achieve low correlations among different waveforms. Array signal processing improves the separability of FM signals impinging from different directions or, in the situation of multipath propagation, with different channel coefficients. FM signals are also used as smart jamming sources. In particular, for FM jammers encountered in direct-sequence spread-spectrum (DS/SS) communications and GPS, TF-based techniques enable improved DOA estimation of the jammers and thereby facilitate their mitigation [27].

In radar applications, LFM signals are commonly used as sensing waveforms, whereas target Doppler signatures demonstrate nonstationarity over the slow time samples. Therefore, TF-based source separation and localization are critical means to handle such signals, particularly when the signals are noisy. Two examples are highlighted below.

- In [28], the STFD-based BSS technique [1] is applied to ground penetration radar for the detection of permafrost interface. Because of the shallow depth, the response from the targets (permafrost) and the clutter from ground surface overlap in the time domain and cannot be separated by simple gating. Spatial filtering is difficult to be applied as well for clutter suppression because the medium is not homogeneous. BSS is considered effective for the separation of target response from ground surface clutter. The nonstationarity of the target response comes from the fact that the response is a superposition of many permafrost scatterers which have different arrival time and different frequencies The measurement data are collected in multiple positions, yielding a synthetic array aperture. As such, the STFD platform is very suited in this problem to exploit the distinguished in time, frequency, and space for effective separation of the permafrost surface response from ground surface clutter. Enhanced identification of permafrost surface is achieved from the separated permafrost response signal.
- A practical and tangible method for providing the altitude information in over the Horizon radar system is based on micro-multipath model that makes use of multipath returns due to the ocean or ground reflections local to the target [29]. As shown in Fig. 6, one path from the target is reflected only by the ionosphere, whereas the other is reflected by the earth surface and the ionosphere. The two paths have different Doppler frequencies when a target maneuvers with an elevation-direction component. As such, the observed multipath received signal corresponding to a maneuvering target becomes a multi-component FM signals with very close IF and angular separations. DOA estimation is possible only after separating the two paths exploiting their difference in the TFD, as each individual signal is converted into a stationary signal and properly filtered [22]. The resolved DOA

estimates of the two paths yield accurate estimation of the target altitude.

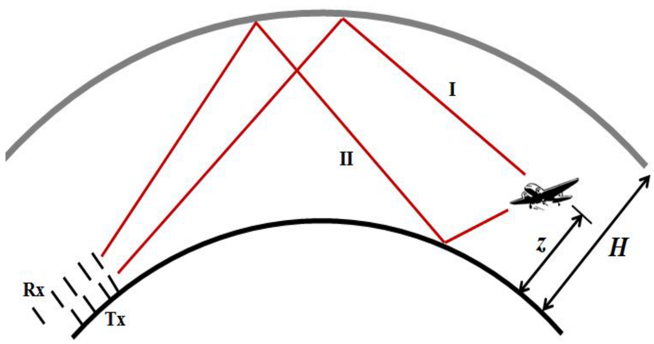


Figure 6. Micro-multipath propagation in an over-the-horizon radar system.

In an underwater environment, LFM signals are also commonly used in active sonar systems, whereas mammals like dolphins rely heavily on sound production and reception to navigate, communicate, hunt, and avoid predators in dark or limited vision waters. In addition, continuous-wave (CW) and narrowband signals are usually distorted due to nonlinear propagation. As such, TF-based array signal processing can play an important role for the separation and localization of emitters. In [30], the TF-MUSIC technique is used in an array of multiple hydrophones to produce passive acoustic oceanic tomography.

The ubiquitous use of multiple sensors, whether in co-located or in distributed architectures, in radar, acoustic and biomedical application areas will continue to invite STFDs to play a role in array signal processing of nonstationary signals. Signals which are instantaneously narrowband are locally sparse by the virtue of their TF power localizations. As such, compressive sensing theory and sparse signal reconstruction can prove effective in achieving source separation and direction finding using significantly reduced number of observations.

The choice of applying linear TF transforms versus quadratic or higher order TFDs in conjunction with array processing will remain to be application specific. It will be influenced by the underlying physical model and prior knowledge of signal characteristics.

VII. Conclusion

In this paper, we presented a review of the spatial time-frequency distributions which constitute an effective framework that enables the integration of time-frequency analysis and array signal processing. Its objective is to use the signal power localization properties in the TF domain to enhance the attributes of multi-sensor receivers, especially for direction finding and source separation of far field nonstationary sources. The key is to permit linear problem formulation where the sensor data are expressed in terms of the source time-frequency signatures. In so doing, SNR enhancement and source discrimination can be exploited prior to performing subspace decomposition and source signal recovery. The paper discussed challenges and approaches for the selection of time-frequency points along the TF signatures as well as at cross-terms. It was emphasized in the paper that the time-frequency and spatial degrees of freedom can interplay and be exploited to enhance their respective domain source characterization. With progress in sensing technology driving lower cost and higher efficiency sensors and with nonstationarity underlying many signal and propagation channel characteristics in emerging applications, it is expected that the STFD framework and others combining power localizations in space, time, and frequency would play a stronger role in signal analysis and sensor data processing.

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