Fast Iterative Interpolated Beamforming for Accurate Single Snapshot DOA Estimation

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***Abstract*—A single snapshot fast computational Fourier-based direction-of-arrival (DOA) estimation method is introduced. This method applies the fast Fourier transform (FFT) to sensor data and performs effective cancellation of spectral leakage caused by sidelobe interactions, leading to unbiased DOA estimates of multiple sources. Successful elimination of spectral leakage is achieved by a sequential removal of strong sinc functions in the spatial frequency domain through an iterative interpolation process. Simulation results demonstrate superior performance of the proposed method over beamforming and other iterative FFTbased DOA estimation techniques, as well as the high-resolution Root-MUSIC algorithm.**

***Index Terms*—Direction-of-arrival (DOA) estimation, single snapshot, fast Fourier transform (FFT), fast iterative interpolation.**

I. Introduction

The problem of single-snapshot, multiple source DOA estimation arises in many radar applications, [2], [3], [4], [5]. Conventional beamforming, though is able to handle both cases of coherent and independent sources, suffers from DOA estimation bias, irrespective of the employed spatial tapering. This bias is caused by spectral leakage through sidelobe interactions of strong or adjacent sources. Reduction in DOA estimation bias can be achieved through a correction term, which has only been derived in the specific case of two sources, [6]. A more effective mitigation of the bias requires iterative schemes, [7], [8], [9]. In the CLEAN algorithm [7], [10], [11], the strongest source is estimated and removed in each iteration. Improvement over the CLEAN algorithm is obtained by refining the source DOA estimates through repeating the iterative process, which describes the RELAX algorithm [9].

The RELAX algorithm iteratively refines the DOA estimates of a growing subset of sources until all sources are obtained. In so doing, it requires FFT computations for each source refinement. In addition, the performance of the estimator is limited by the coarseness of the spatial frequency grid, which necessitates sufficient zero-padding to achieve the desired accuracy. We adapt a recently proposed fast iterative multi-component frequency estimator [12], [13] to beamforming for the single snapshot multi-source DOA estimation problem. We refer to the proposed technique as Fast Iterative Interpolated Beamformer (FIIB). Unlike RELAX, FIIB only requires a single FFT calculation on a coarse grid combined with interpolation using a small number of additional Fourier coefficients. The DOA estimates are refined over successive cycles; within each cycle, all sources are sequentially estimated. The bias in the DOA estimates is eliminated by incorporating a leakage subtraction step into the iterations.

The FIIB has desirable convergence properties, which we exploit by proposing an effective stopping criterion based on an adaptive tolerance level. It has a computational complexity of the same order as the FFT. It is shown to achieve excellent DOA estimation performance even for components that are widely separated in power. We demonstrate that the proposed method outperforms both RELAX and the high-resolution Root-MUSIC method, [14].

The paper is organized as follows. In Section II, we present the signal model. In Section III, existing FFT-based DOA estimation techniques are briefly reviewed. Section IV describes the proposed algorithm, followed by a discussion on its convergence and computational cost in Section V. Simulation results are presented in Section VI and finally some conclusions are drawn in Section VII.

II. Signal Model

Consider a linear array of M equispaced sensors with inter-element spacing, d, measured in wavelengths. The signals of L narrowband point sources impinge on the array from directions $\theta_\ell,\ \ell = 1, \dots, L$. The $M \times 1$ single snapshot vector of signals at the output of the array is given by

$$\mathbf{x} = \sum_{\ell=1}^{L} \alpha_\ell \mathbf{a}(\theta_\ell) + \mathbf{n}, \tag{1}$$

where α_ℓ is the complex amplitude of the ℓ-th source, $\mathbf{n}$ is the $M \times 1$ vector of additive Gaussian noise with zero mean and covariance $\sigma_n^2 \mathbf{I}_M$, and $\mathbf{I}_M$ is the $M \times M$ identity matrix. The steering vector, $\mathbf{a}(\theta_\ell)$, of the ℓ-th source is defined as

$$\mathbf{a}(\theta_\ell) = \left[1, e^{-j2\pi\nu(\theta_\ell)}, \dots, e^{-j2\pi(M-1)\nu(\theta_\ell)}\right]^T, \tag{2}$$

where $\nu(\theta) \triangleq d\sin(\theta) \in [-\frac{1}{2}, \frac{1}{2}]$ is the spatial frequency associated with direction θ and $(\cdot)^T$ stands for transpose. For simplicity of notation, we set $\nu_\ell \equiv \nu(\theta_\ell)$. The signal to noise ratio (SNR) of the ℓ-th source is defined as $\rho_\ell = 10\log_{10}\frac{|\alpha_\ell|^2}{\sigma_n^2}$.

We assume that the number of sources, L, is known a priori or through the use of an information-theoretic measure, [9]. Our objective is to estimate the DOAs $\theta_\ell,\ \ell = 1, \dots, L$ by

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Part of the results were presented at the 2016 IEEE Radar Conference [1].

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estimating ν_ℓ in a computationally efficient manner. In the sequel, we present a new method for single-snapshot DOA estimation using fast iterative interpolated beamforming.

The multi-source Cramér-Rao Bound (CRB) for DOA estimation is given by the inverse of the Fisher information matrix [15] and its expression is not reproduced here. In the single source case, the CRB of the spatial frequency $\nu(\theta)$ reduces to

$$\mathrm{CRB}(\nu) = \frac{6}{(2\pi)^2 \rho M (M^2 - 1)}, \tag{3}$$

where ρ is the SNR. The CRB for azimuth angle, θ is

$$\mathrm{CRB}(\theta) = \frac{\mathrm{CRB}(\nu)}{d^2 \cos^2(\theta)} = \frac{\mathrm{CRB}(\nu)}{d^2 \cos^2(\sin^{-1} \nu)}. \tag{4}$$

III. Existing FFT-Based DOA Estimation Techniques

In this section, we review two popular FFT-based DOA estimation methods: the celebrated conventional beamformer (BF) and the RELAX algorithm.

A. Conventional Beamformer Based DOA Estimation

In essence, the conventional BF steers the array response towards the spatial direction that maximizes the output power [3]. In the single snapshot case, the BF spectrum is defined as

$$P_{\mathrm{BF}}(\theta) = \left| \mathbf{a}^H(\theta)\mathbf{x} \right|^2. \tag{5}$$

Given L, the DOAs can be estimated by locating the L highest peaks of the BF spectrum. For the special case of a single source, the BF-based estimator (BFE) simplifies to the deterministic maximum likelihood estimator [3]. But, this requires that (5) is computed on an infinite number of grid points.

In practice, the BFE is implemented efficiently using the FFT. Let $X^{(z)}[k]$ be the discrete Fourier transform (DFT) of $\mathbf{x}$ evaluated on a uniform grid of $K = zM$ points, that is

$$X^{(z)}[k] = \mathrm{FFT}\big(\mathbf{x}, K\big), \quad k = 0, 1, \ldots, K-1, \tag{6}$$

where the integer $z \geq 1$ defines the amount of zero padding required to compute a K-point FFT. The DOA estimates are then given by the L highest peaks of the discrete spectrum

$$P_{\mathrm{BF}}[k] = \left| X^{(z)}[k] \right|^2. \tag{7}$$

For high accuracy, the discrete spectrum (7) should be calculated using large values of z. In fact, for an SNR ρ, the required $z \propto \sqrt{M\rho}$ can grow quite large with M and ρ. It is worth noting that, for the multiple source case, the BF based DOA estimator suffers from an estimation bias. Extensive research efforts to address this problem have traditionally focused on reducing the bias by pre-windowing the signal, [16], [17]. This approach, however, comes at the cost of higher variance of the estimates. In [6], a simple method for bias reduction, specific to the case $L = 2$ sources, was developed. But, this method does not eliminate the bias, neither does it apply to the $L > 2$ case.

B. RELAX Based DOA Estimation

The RELAX algorithm was developed in [9] for sinusoidal parameter estimation and modified in [18] for applications to sensor arrays. It estimates the source DOAs via finding an iterative solution to the following nonlinear problem

$$(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\nu}}) = \arg\min_{\boldsymbol{\alpha}, \boldsymbol{\nu}} \left\| \mathbf{x} - \sum_{\ell=1}^{L} \alpha_\ell \mathbf{a}(\theta_\ell) \right\|^2, \tag{8}$$

where $\boldsymbol{\alpha} \triangleq [\alpha_1, \ldots, \alpha_L]^T$ and $\boldsymbol{\nu} \triangleq [\nu_1, \ldots, \nu_L]^T$. Instead of performing an exhaustive multi-dimensional search, RELAX solves (8) sequentially over L cycles. During the J-th ($1 \leq J \leq L$) cycle, the algorithm iteratively estimates the parameters of the J strongest sources, where the estimates from the $(J-1)$-th cycle are used as initial estimates. The signal associated with the ℓ-th source is obtained as

$$\mathbf{x}_\ell = \mathbf{x} - \sum_{i=1, i\neq\ell}^{J} \alpha_i \mathbf{a}(\theta_i). \tag{9}$$

For each of the source signals in (9), the single-source problem is solved via the BF method with a zero-padded FFT, that is,

$$X_\ell^{(z)}[k] = \mathrm{FFT}\big(\mathbf{x}_\ell, K\big), \quad k = 0, 1, \ldots, K-1. \tag{10}$$

It is worth noting that (10) needs to be calculated for every source during each iteration over all L cycles. More details on the implementation of RELAX can be found in [18].

IV. Proposed Fast Iterative Interpolated Beamforming Estimator

The proposed FIIB technique is non-parametric and is based on the FFT to achieve unbiased and accurate estimation of multiple source DOAs. Like many approaches in the literature, our technique uses an estimate-and-subtract strategy to successively extract the sources in an inner loop. This inner loop is then wrapped in an outer loop that allows the estimates to be refined in order to eliminate the bias at convergence. At the heart of the algorithm is a simple yet highly accurate interpolation strategy that is combined with a leakage subtraction scheme. This combination endows the algorithm with excellent convergence properties. Unlike RELAX, the proposed technique does not require any zero-padding and is capable of tracking the CRB at all SNRs, making it an effective approach for Fourier-based estimation in the multi-source case.

The FIIB algorithm is given in table I. The conventional beamforming coefficients $X[n]$ are obtained using the K-length FFT, for small $z = 1$ or 2. The subsequent processing is carried out in the frequency domain, avoiding the need to re-use the FFT. In the first iteration, we obtain coarse estimates of the L DOAs sequentially, starting with the strongest source. Specifically, for the ℓ-th source, we first subtract the previously estimated sources from the signal as shown in Step (i) and then locate the highest peak of the spectrum in Step (ii). This ensures that the previously estimated ℓ–1 sources are removed, exposing the ℓ-th source. Note that $\hat{S}_i[n]$ in Step (i) is the DFT coefficient of the i-th source steering vector at the frequency $\nu = \frac{n}{K}$. That is $\hat{S}_i[n] = \hat{S}_i(\nu) \mid_{\nu=\frac{n}{K}}$ where

$$\hat{S}_i(\nu)=\sum_{k=0}^{M-1} e^{j2\pi k\hat{\nu}_i} e^{-j2\pi\nu} = \frac{1-e^{j2\pi M(\hat{\nu}_i-\nu)}}{1-e^{j2\pi(\hat{\nu}_i-\nu)}}. \tag{11}$$

The coarse estimate of the ℓ-th source is then refined using interpolation on the Fourier coefficients in steps (iii) – (vi).

Fourier-based interpolation methods have achieved a high degree of maturity in single harmonic frequency estimation [19], [20]. In the multiple signal case, on the other hand, they have been much less appealing due to inherent bias induced by spectral leakage. Combining an interpolation strategy with the successive estimate-and-subtract approach has the potential to eliminate the bias and give accurate estimates, provided the interpolation function has suitable convergence properties when implemented iteratively. These exact properties are offered by the interpolator of [21]. To refine the DOA estimate $\hat{\nu}_\ell$, we calculate in Step (iii) two new leakage-corrected DFT coefficients that are $p = \pm\frac{z}{2}$ bins away from $\hat{\nu}_\ell$. That is,

$$\begin{aligned}\tilde{X}_p(\hat{\nu}_\ell) &\equiv \tilde{X}(\hat{\nu}_\ell + p)\\ &= X(\hat{\nu}_\ell+p) - \sum_{i=1,i\neq\ell}^{L} \hat{\alpha}_\ell \hat{S}_i(\hat{\nu}_\ell+p),\end{aligned} \tag{12}$$

where (11) is used to express the leakage DFT coefficients as

$$\hat{S}_i(\hat{\nu}_\ell+p) = \frac{1+e^{j2\pi M(\hat{\nu}_i-\hat{\nu}_\ell)}}{1-e^{j2\pi\left(\hat{\nu}_i-\hat{\nu}_\ell-\frac{p}{K}\right)}}. \tag{13}$$

The leakage-free coefficients of (12) are then used in Step (v) to obtain the discriminant function h, and the frequency and amplitude estimates are updated in Steps (vi) and (vii). Here, Re[•] defines the real part of •. This procedure is executed for all sources (inner loop) and the algorithm is run for Q iterations or until convergence (outer loop). The algorithm is summarised in the following sequence for each source: 1) subtract all other sources (Step i), 2) in first iteration find coarse estimate (Step ii), 3) refine the DOA estimate (Steps iii-vi), and 4) Obtain the complex amplitude estimate (Step vii). Finally, the DOA of source ℓ is calculated as $\hat{\theta}_\ell = \sin^{-1}(\hat{\nu}_\ell/d)$.

V. Convergence and Computational Complexity

In the single source case, the performance of the estimator is only affected by the noise, and the interpolation converges in 2 iterations [21]. Convergence is defined as the point where the distance between the estimate and the true frequency is of lower order than the CRB. In the multi-source case, the iterative procedure was shown in [22] to converge to the fixed point of the iteration which coincides with the true frequencies. The convergence rate was also derived and found to depend on the interplay between leakage and noise. We observe two cases. If the maximum leakage is smaller than the noise, which occurs at sufficiently low SNR, then convergence is dictated by the noise and is achieved only in two iterations. If the leakage is stronger than the noise, it gets reduced by the algorithm below the noise level, at which point only one additional iteration is required for convergence. At convergence, the algorithm is unbiased and the estimation variance is of the same order as the CRB.

TABLE I
The Proposed FIIB Estimator

Initialization:
Put $\hat{\nu}_\ell = 0$ and $\hat{\alpha}_\ell = 0$ for $\ell = 1\ldots L$
Let $X \equiv FFT(x, K)$, with $z = 1$ or 2, and $K = zM$
Set $q = 0$
Loop until Q iterations or convergence: (Outer Loop)
$q \leftarrow q + 1$
For $\ell = 1\ldots L$ do (Inner Loop)
if $q == 1$ then

$$\tilde{X}[n] = X[n] - \sum_{i=1,i\neq\ell}^{L} \hat{\alpha}_i \hat{S}_i[n],\ n = 0\ldots K-1 \tag{i}$$

$$\hat{\nu}_\ell = \frac{1}{K} \arg\max_{1\leq n\leq K} |\tilde{X}[n]|^2 \tag{ii}$$

Fine Frequency Estimation:

$$\tilde{X}_{\pm p}(\hat{\nu}_\ell) = X_{\pm p}(\hat{\nu}_\ell) - \sum_{i=1,i\neq\ell}^{I} \hat{\alpha}_i \hat{S}_i(\hat{\nu}_\ell \pm p),\ p = \frac{z}{2} \tag{iii}$$

$$\text{where} \quad X_p(\hat{\nu}_\ell) \equiv X(\hat{\nu}_\ell + p) \tag{iv}$$

$$h = \mathrm{Re}\left[\frac{\tilde{X}_p(\hat{\nu}_\ell)+\tilde{X}_{-p}(\hat{\nu}_\ell)}{\tilde{X}_p(\hat{\nu}_\ell)-\tilde{X}_{-p}(\hat{\nu}_\ell)}\right] \tag{v}$$

$$\hat{\nu}_\ell \leftarrow \hat{\nu}_\ell + \frac{h}{2K} \tag{vi}$$

$$\hat{\alpha}_l = \frac{1}{M}\left\{\sum_{k=0}^{M-1} x[k]e^{-j\frac{2\pi}{K}k\hat{\nu}_\ell} - \sum_{i=1,i\neq\ell}^{L} \hat{\alpha}_i \hat{S}_i(\hat{\nu}_l)\right\} \tag{vii}$$

Output: $\hat{\nu}_\ell \leftarrow \frac{\hat{\nu}_\ell}{K}$ for $\ell = 1\ldots L$

In [1], it was suggested that the algorithm should run until the maximum difference between two successive frequency estimates is less than a specified tolerance. The optimal tolerance value, however, depends on the SNR and number of antennas M, and setting it is not a trivial matter. A value set too large could fail to eliminate the estimation bias, while a small value can incur a higher than necessary computational burden. Thus, we propose an adaptive approach where the tolerance of the ℓ-th source is set adaptively, depending on its CRB, that $\tau_\ell = \mathrm{CRB}(\hat{\nu}_\ell)$. This strategy ensures that a component with high SNR has a small tolerance value in line with its CRB, but it requires the calculation of the CRB at every iteration. Since the single source CRB lower bounds the corresponding multi-source CRB, we adopt the single source CRB as given by (3). In this equation, the source SNR is obtained from the estimated source ampliude and an estimate of the noise power.

Let $\{\hat{\nu}_\ell^{(q)}, \hat{\alpha}_\ell^{(q)}\}_{1\leq\ell\leq L}$ be the frequency and amplitude estimates at iteration q. The following calculations are carried out after Step (vii) in Table I:

1) Estimate the noise power using the mean residual power:

$$\hat{\sigma}^2 = \frac{1}{M}\left\|\mathbf{x} - \sum_{\ell=\mathbf{1}}^{\mathbf{L}} \hat{\alpha}_\ell \mathbf{a}(\hat{\nu}_\ell)\right\|^2. \tag{14}$$

2) For the ℓ-th source, obtain an estimate of the SNR as $\hat{\rho}_\ell = \frac{|\hat{\alpha}_\ell|^2}{\hat{\sigma}^2}$ and then calculate the tolerance τ_ℓ using (3).
3) If $\left|\hat{\nu}_\ell^{(q)} - \hat{\nu}_\ell^{(q-1)}\right|^2 \leq \tau_\ell\ \forall\ 1 \leq \ell \leq L$, then stop.

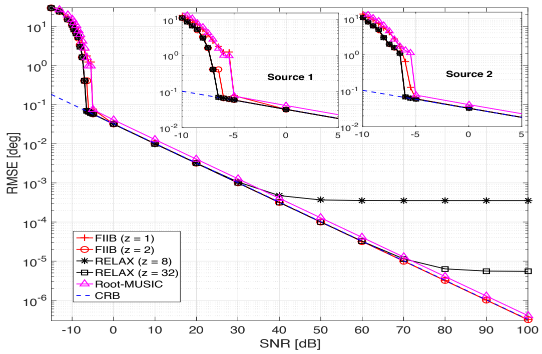

Fig. 1. RMSE performance as a function of SNR for Source 1 (the Source 2 result is identical).

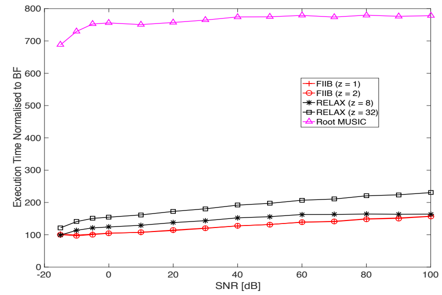

Fig. 2. Ratio of execution time of the proposed and RELAX algorithms to the conventional BF versus SNR.

Now, the estimator requires the calculation of the FFT only once at a computational cost of $O\{M \log_2 M\}$. Then, two new DFT coefficient are used per source in each iteration, which requires $O\{2LM\}$ calculations. Consequently, the overall complexity for Q iterations is $O\{M \log_2 M + Q(2LM + \kappa)\}$, where κ is a constant that accounts for additional overhead per iteration (including the calculation of the tolerance).

For comparison, the RELAX algorithm requires a zero-padded FFT with a dense grid corresponding to a large z. The corresponding interval of the search grid is $\Delta = 2\pi/K$ measured in radians. At high SNR, the accuracy of the estimates is limited by the bias attributed to the finite grid size. This error is uniformly distributed over the interval $[-\Delta/2, \Delta/2]$ and has variance of $\Delta^2/12$. Let the CRB value at the operational SNR be C. To achieve the CRB, the grid should be sufficiently dense such that $\frac{\Delta^2}{12} \leq C$. This implies that $K \geq \frac{\pi}{\sqrt{3C}} \propto \sqrt{\rho M^3}$. For Q iterations, RELAX requires approximately $QLK \log_2 K$ computations. For large M, this amounts to $O\{M^{\frac{3}{2}} \log_2 M\}$, which grows faster than the complexity of the proposed method. It is important to note that the computational cost of RELAX also increases with increasing SNR.

VI. Simulation Results

Consider a uniform linear array (ULA) comprising $M = 128$ antennas with half-wavelength spacing. Signals originating from sources located in the far-field impinge on the ULA from directions θ_ℓ with corresponding spatial frequencies $\nu_\ell = 0.5 \sin(\theta_\ell)$. The source magnitudes, $|\alpha_\ell|$, are set according to the desired SNRs and their phases are uniformly drawn from the interval $[0,\ 2\pi]$. We model the noise as zero-mean Gaussian with variance $\sigma_n^2 = 1$. In all examples, we use $5,000$ independent runs and convert the spatial frequencies to DOAs in degrees. We compare the proposed method to the RELAX and Root-MUSIC algorithms (using the optimized MATLAB™ implementation of Root-MUSIC). Note that we adopt for a subarray size of $\frac{M}{2} - 1$ for Root-MUSIC.

Two-Source DOA Estimation Performance Versus SNR

In this example, we examine the estimation performance vs. SNR for two sources. We set $|\alpha_1| = |\alpha_2|$, and the electrical angle separation between them to $1.8BW$, where the bin width $BW = \frac{2\pi}{M}$. We implement RELAX for two zero-padding factors, $z = 8$ and $z = 32$, and FIIB for $z = 1$ and $z = 2$.

Fig. 1 gives the RMSEs versus SNR for Source 1 (the curves for source 2 are very similar as shown in the insets). The results confirm that the performance of RELAX coincides with the CRB at moderate SNR but deviates from it at high SNR values with the RMSE saturating as a result of performing the FFT on a finite grid. This variance floor can be reduced by increasing the zero-padding factor, i.e., using a denser grid, but this comes at the price of increased computational load. It is clear from the figure that the proposed FIIB algorithm achieves the CRB at all SNR values. It outperforms RELAX at high SNR and is slightly better than Root-MUSIC at all SNRs. Also, FIIB has a better threshold than Root-MUSIC. As the threshold performance is affected by the amount of zero-padding, RELAX naturally has a slightly better breakdown threshold than the other algorithms.

In Fig. 2 we present the ratio of the execution times of the algorithms to the FFT. The superiority of the proposed algorithm in this respect is evident. Interestingly, the introduction of the adaptive tolerance leads to a reduction in computational cost for the proposed estimator, but an increase in that of the RELAX algorithm. This is due to the fact that RELAX cannot converge owing to the bias under finite grid size, leading to a variance floor. FIIB, on the other hand, requires fewer iterations on average as it rapidly converges. We point out that doubling the zero-padding does not incur any significant increase in computational cost. Finally, notice that Root-MUSIC is much more computationally expensive that the other two algorithms. The variation of its execution time is due to the calculation of the eigenvalues. This would explain why there is a slight reduction in execution time as the SNR drops below the threshold (which can be seen in Fig. 4 to be around -5dB). As the SNR drops, the noise dominates and the covariance matrix tends towards the diagonal.

Performance As a Function of Source Separation

In the second example, we examine the performance as a function of the source separation. We use two sources with equal powers and set the SNR equal to 30dB. Without loss of generality, we take the phase of Source 1 to be 0, whereas that of the second source is drawn uniformly from the interval $[0, 2\pi]$. We vary the source separation from, a sub-bin

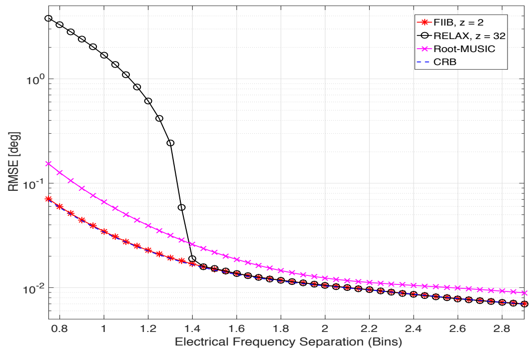

Fig. 3. RMSE performance of Source 1 versus the electrical angle separation between the two sources.

resolution, $0.75BW$ to $2.9BW$. We compare, in Fig. 3, the FIIB (with $z = 2$) to RELAX ($z = 32$) and Root-MUSIC. First, notice that although FIIB is an FFT-based estimator, it has high-resolution capability. In fact, it outperforms both RELAX and Root-MUSIC at all simulated source separations.

A. Three Sources with Unequal Powers

We now demonstrate the performance of FIIB in the case of three sources with unequal powers. This scenario may be encountered in many practical applications, such as radar-assisted smart automotive systems where the target of interest is observed against a background of multipath reflections and/or interference from other vehicles. We choose the DOA of Source 2 randomly and set the DOAs of Sources 1 and 3, respectively, at $1.8BW$ to the left and $2.3BW$ to the right of source 2. Additionally, we take the power of Sources 1 and 3 to be 5dB and 10 dB respectively below that of Source 2.

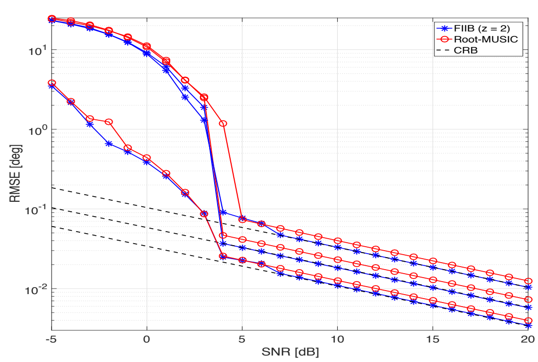

Fig. 4. RMSE vs SNR for a three-source case. The bottom set of curves belong to Source 2, the middle set to Source 1, and the top set to Source 3.

Fig. 4 shows the RMSEs of the three sources. We only display FIIB and Root-MUSIC, since RELAX performs worse than the other two. We see that FIIB achieves the CRB and has a lower RMSE than Root-MUSIC. Also, the SNR differential between the three sources conforms to the chosen power ratios.

VII. Conclusions

We have examined the problem of DOA estimation of multiple sources in noise. We proposed a new, high fidelity estimation algorithm that enjoys the same computational complexity as the FFT, is capable of delivering unbiased DOA estimates, and achieves the CRB for each of the sources individually. We presented simulation results demonstrating the superiority of the proposed estimator over the well-known RELAX and Root-MUSIC algorithms in terms of accuracy, resolution and speed.

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