

Analysis of Communication Symbol Embedding in FH MIMO Radar Platforms

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Abstract—Information embedding into the emission of multiple-input multiple-output (MIMO) frequency hopping (FH) radar is analyzed. It is assumed that the radar is primary under dual function radar communication system platforms. Phase shift keying (PSK) communication symbols are embedded in each hop of the FH waveforms. The impact of embedding is a significant reduction in range sidelobe levels. We examine the impact of PSK symbol embedding on the power spectral density (PSD) of the MIMO radar. It is shown that there is spectral leakage due to the disruption of the continuous phase of the FH waveforms by the PSK symbol embedding. The trade-off between low sidelobe levels and low spectral leakage is analyzed. To maintain the phase continuity between the frequency hops, modulation of FH waveforms with frequency shift keying (FSK) symbols is considered and its performance is compared with that of the FH/PSK radar communication waveforms.

I. INTRODUCTION

Increase in the demands of the commercial wireless communications industry has created radio frequency (RF) spectrum congestion challenges, leading to strong interest in spectrum sharing [1], [2]. Spectrum sharing research has been categorized into, co-habitation and co-design [3]–[7]. In the co-habitation approach, radar and communication co-exist as separate systems adapting and responding to each other existence. The co-design approach, in principle, avoids any frequency contentions by signal design, dynamic frequency allocation, or through a dual functional system approach. In the latter, both services are launched from the same system, sharing platform resources such as waveform, power, bandwidth, and aperture [8]–[13]. When communications is treated as secondary to the primary radar function, the system is referred to as dual-function radar-communication (DFRC) [14]–[20]. DFRC systems make full use of the radar resources such as high quality hardware and high transmit power.

For the DFRC system, information embedding into the emission of single-input multiple-output (SIMO) radar can be achieved using waveform diversity, sidelobe control, or time modulated array. Information embedding into the emission of multiple-input multiple-output (MIMO) radar has been considered in [21], [22]. Alternative names used in the literature for the DFRC system are Intentional Modulation on a Pulse and CoRadar [23], [24].

In this paper, we show that communication signal embedding can benefit dual system functionalities. We use the waveform design approach in [19] where information embedding is performed using frequency-hopping (FH) waveforms in MIMO radar. We assume a single antenna communication receiver that is phase and time synchronized with the DFRC transmitter. The impact of embedding phase shift keying (PSK) communication symbols on the MIMO radar functionality is examined. This impact is measured by analyzing the ambiguity function (AF) characteristics pre- and post- signal embedding. The nominal FH-based MIMO radar can exhibit large sidelobes due to the re-use of the FH coefficients within the same pulse [25]. It is shown that PSK communication symbol embedding considerably reduces the sidelobe level (SLL) of the corresponding AF. As a result, a large number of orthogonal FH waveforms can be synthesized via increasing the rate of FH coefficient recurrence. This, in turn, increases the achievable communication data rate without altering or compromising the MIMO radar functionality.

Due to the PSK symbol embedding in each hop of the FH radar waveforms, there would be phase discontinuity between the hops. This PSK symbol embedding disrupts the continuous phase of the FH waveforms. As a result, the spectral sidelobes become more pronounced leading to spectral leakage into the side bands. This adverse effect can be mitigated by embedding zero phase PSK symbols in the lower and higher hop frequencies in the allocated bandwidth. We analyze the tradeoff between low SLLs of AF and low spectral leakage. An alternative embedding approach to preserve the continuous phase of the FH waveforms is to modulate the FH waveforms by the frequency shift keying (FSK) symbols. The performance of the FH/FSK modulation scheme is investigated and contrasted with that of the FH/PSK modulation.

The paper is organized as follows. The details of the DFRC system design, the FH waveforms, PSK symbol embedding, bandwidth requirements and the MIMO radar signal models are presented in Sec. II. Analysis of the FH/PSK waveform design is presented in Sec. III. FH/FSK waveforms are presented and analyzed in Sec. IV. Simulation results are given in Sec. V and conclusions are drawn in Sec. VI.

II. FH MIMO RADAR SIGNAL MODEL

We consider a system equipped with a common dual-function transmit platform. The common transmit array comprises M omni-directional co-located transmit antennas and the MIMO radar receive array comprises N antennas arranged in a linear shape.

A. Frequency-Hopping Waveforms

FH waveforms are inherently power efficient due to the constant-modulus property. They are also simple to generate and have the advantage of low probability of intercept (LPI) as the transmit hop sequence is not known to the intercept receiver. FH waveforms meet the MIMO radar requirements, like high transmit power efficiency, high range and Doppler resolution properties. In this respect, the use of FH waveforms for MIMO radar has been reported in a number of papers [20], [26]. The m^{th} antenna FH waveform can be expressed as

$$\phi_m(t) = \sum_{q=1}^Q e^{j2\pi c_{m,q} \Delta_f t} u(t - q\Delta_t) \quad (1)$$

where $c_{m,q}$, $m = 1, \dots, M$, $q = 1, \dots, Q$ denote the FH coefficients, Q is the FH code length, Δ_f and Δ_t are the frequency step and the sub pulse duration, respectively, and

$$u(t) \triangleq \begin{cases} 1, & 0 < t < \Delta_t, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

is a rectangular pulse of duration Δ_t . Equation (1) implies that each FH waveform contains Q sub-pulses, i.e., the pulse duration is $T_p = Q\Delta_t$. It is also assumed that $\Delta_t \Delta_f$ is an integer.

FH coefficients ($c_{m,q}$) play an important role in designing the FH waveforms. To enable symbol detection at the communication receiver, an additional constraint to maintain the orthogonality between the FH waveforms from sub-pulse to another across antennas at zero delay ($\tau = 0$), is mandated by the communication function of the system [20], [26].

$$c_{m,q} \neq c_{m',q}, \quad \forall q, m \neq m' \quad (3)$$

The above condition implies that for a given sub-pulse duration, no two frequencies should be same. This condition, however, does not restrict the reuse of frequencies over different hops for a given antenna. In this paper, we generate the FH code for each sub-pulse using an iterative random frequency generation so as the above condition is satisfied. In the case when $c_{m,q}$ is used only once in the FH waveform generation, the number of antennas and hops that can be used will be limited by the condition $MQ = K$, where K is the maximum number of frequencies available for the operating bandwidth. This condition, in effect, reduces the number of antennas and hops that can be employed by the system which translates to a reduced data rate.

B. PSK Symbol Embedding

This section provides an overview of PSK symbol embedding into the emission of MIMO radar using FH waveforms. For more information on the PSK signaling scheme, the reader is referred to [19], [27].

Let $\{\Omega_{(m,q)}^{(n)} \in \mathbb{D}_{\text{PSK}}, m = 1, \dots, M, q = 1, \dots, Q$ be a set of PSK symbols that need to be embedded into the MIMO radar emission during the n^{th} pulse, where $\mathbb{D}_{\text{PSK}} = \{0, \frac{2\pi}{J}, \dots, \frac{(J-1)2\pi}{J}\}$ is a PSK dictionary of size J . The PSK symbols are positioned on a complex unit circle. Each PSK symbol represents $N_{\text{bit}} = \log_2 J$ bits. Accordingly, the PSK-modulated FH radar waveforms are given by

$$\psi_m(t, n) = \sum_{q=1}^Q e^{j\Omega_{(m,q)}^{(n)} h_{m,q}(t)} u(t - q\Delta_t - nT_0), \quad (4)$$

where $h_{m,q}(t) \triangleq e^{j2\pi c_{m,q} \Delta_f t}$ is the FH signal associated with the m^{th} antenna during the q^{th} sub-pulse and T_0 is the pulse repetition interval (PRI).

C. MIMO Radar Receive Signal Model

Assume that the signals reflected by L targets impinge on the MIMO radar receiver from directions θ_ℓ , $\ell = 1, \dots, L$. We consider the signal model given in [28] to express the $N \times 1$ complex-valued vector of the received baseband signals,

$$\mathbf{x}(t, n) = \sum_{\ell=1}^L \beta_\ell(n) [\mathbf{a}^T(\theta_\ell) \boldsymbol{\psi}(t, n)] \mathbf{b}(\theta_\ell) + \mathbf{z}(t, n), \quad (5)$$

where n denotes the slow-time index, i.e., pulse number, $\beta_\ell(n)$ is the reflection coefficient associated with the ℓ^{th} target during the n^{th} pulse, θ_ℓ is the spatial angle of the ℓ^{th} target, $\mathbf{a}(\theta_\ell)$ and $\mathbf{b}(\theta_\ell)$ are the steering vectors of the transmit and receive arrays towards the direction θ_ℓ , respectively, $(\cdot)^T$ stands for the transpose, $\boldsymbol{\psi}(t, n) \triangleq [\psi_1(t, n), \dots, \psi_M(t, n)]^T$ is the $M \times 1$ complex vector of PSK modulated waveforms, and $\mathbf{z}(t, n)$ is an $N \times 1$ vector of zero-mean white Gaussian noise with variance σ_z^2 . Matched filtering (5) to the transmitted orthogonal waveforms yields the $MN \times 1$ data vector

$$\begin{aligned} \mathbf{y}(n) &= \text{vec} \left(\int_{T_p} \mathbf{x}(t, n) \boldsymbol{\psi}(t, n)^H(t) dt \right) \\ &= \sum_{\ell=1}^L \beta_\ell(n) [\mathbf{a}(\theta_\ell) \otimes \mathbf{b}(\theta_\ell)] + \tilde{\mathbf{z}}(n), \end{aligned} \quad (6)$$

where $\text{vec}(\cdot)$ denotes the vectorization operator that stacks the columns of a matrix into one long column vector, \otimes denotes the Kronecker product, $(\cdot)^H$ stands for the Hermitian transpose, $\tilde{\mathbf{z}}(n)$ is $MN \times 1$ vector of additive noise at the output of the matched-filters with zero-mean and co-variance $\sigma_z^2 \mathbf{I}_{MN}$, and \mathbf{I}_M is the identity matrix of size $M \times M$.

Consider a single-antenna communication receiver located at in the spatial direction θ_c with respect to the MIMO radar. The signal at the output of the communication receiver is

$$r(t, n) = \alpha_{\text{ch}} \mathbf{a}^T(\theta_c) \boldsymbol{\psi}(t, n) + w(t, n), \quad (7)$$

where α_{ch} is the channel coefficient which summarizes the propagation environment between the transmit array and the communication receiver and $w(t, n)$ represents the additive white Gaussian noise with zero mean and variance σ_w^2 .

Assume that time and phase synchronizations between the MIMO radar and the communication receiver are achieved. Then, matched filtering $r(t, n)$ to the FH sub-pulses yields

$$\begin{aligned} y_{m,q}(n) &= \int_{\Delta_t} r(t, n) h_{m,q}^*(t) u(t - q\Delta_t - nT_0) dt \\ &= \alpha_{\text{ch}} e^{j(\Omega_{p,m}^{(n)} - 2\pi d_m \sin \theta_c)} + w_{m,q}(n), \end{aligned} \quad (8)$$

where d_m is the displacement between the first and the m^{th} elements of the transmit array measured in wavelength, and $w_{m,q}(n) \triangleq \int_0^{\Delta_t} w(t, n) e^{-j2\pi c_{m,q} \Delta_f t} u(t - \Delta_t - nT_0) dt$ is the additive noise term at the output of the $(m, q)^{\text{th}}$ matched filter with zero mean and variance σ_w^2 . Then, the embedded PSK symbols can be estimated as [19]

$$\hat{\Omega}_{(m,q)}^{(n)} = \angle(y_{m,q}(n)) - \varphi_{\text{ch}} + 2\pi d_m \sin \theta_c, \quad (9)$$

where $\angle(\cdot)$ stands for the angle of a complex number and $\varphi_{\text{ch}} \triangleq \angle(\alpha_{\text{ch}})$ is the phase of the channel coefficient.

D. Bandwidth Requirements

Let B be the system bandwidth. To insure that the spectral contents of the FH waveforms are confined to the available bandwidth, the FH code should be selected from the set of integers $\{0, 1, \dots, K-1\}$, where $K \triangleq \frac{B}{\Delta_f}$. Then, the effective bandwidth can be approximated as

$$B_{\text{eff}} \approx (K-1)\Delta_f + \frac{1}{\Delta_t}. \quad (10)$$

We assume that the value of K is properly selected such that the condition $B_{\text{eff}} \leq B$ is satisfied. The time-bandwidth product of the DFRC system is given as

$$BT_p \approx \left((K-1)\Delta_f + \frac{1}{\Delta_t} \right) Q\Delta_t = KQ. \quad (11)$$

III. ANALYSIS OF THE EFFECT OF PSK SYMBOL EMBEDDING ON THE DFRC SYSTEM

A. Effect of PSK symbol embedding on Ambiguity Function

We present the analysis of the Ambiguity Function (AF) [29] to show the benefits of using the PSK symbol embedding. The cross AF expression for the MIMO radar is given by [20]

$$\chi_{m,m'}(\tau, \nu) \triangleq \int_0^{T_p} \phi_m(t) \phi_{m'}^*(t + \tau) e^{j2\pi \nu t} dt \quad (12)$$

We consider the case of a DFRC system with uniform linear arrays. The inter-element spacings associated with the transmit and receive arrays are denoted as d_T and d_R , respectively. The spatial frequency of a hypothetical target located in direction θ is defined as $f_{sp} = 2\pi d_R \sin(\theta)/\lambda$, where λ is the carrier

wavelength. The AF for MIMO radar using FH waveforms is given by,

$$\left| \chi_{\text{radar}}(\tau, \nu, f_{sp}, f'_{sp}) \right| = \left| \sum_{m=1}^M \sum_{m'=1}^M \sum_{q=1}^Q \sum_{q'=1}^Q \chi_{m,m'}^{(\Delta_t)}(\tau, \nu) e^{j2\pi(f_{sp}m - f'_{sp}m')\gamma} \right|, \quad (13)$$

where, $\tau, \nu, f_{sp}, f'_{sp}$ denote time delay, Doppler shift, spatial frequency, and spatial frequency shift, respectively, $\gamma \triangleq d_T/d_R$, and

$$\chi_{m,m'}^{(\Delta_t)}(\tau, \nu) = \chi_{m,m'}^{\text{rect}}(\tau_1, \nu_1) e^{j2\pi c_{m',q'} \Delta_f \tau}, \quad |\tau| \leq \Delta_t \quad (14)$$

$$\begin{aligned} \chi_{m,m'}^{\text{rect}}(\tau_1, \nu_1) &\triangleq \int_0^{\Delta_t} u(t) u(t + \tau_1) e^{j2\pi \nu_1 t} dt, \\ &= \frac{\Delta_t - |\tau_1|}{\Delta_t} \text{sinc}(\nu_1(\Delta_t - |\tau_1|)) e^{j\pi \nu_1(\Delta_t - |\tau_1|)} \end{aligned} \quad (15)$$

In (14), $\nu_1 \triangleq (c_{m,q} - c_{m',q'})\Delta_f + \nu$ and $\tau_1 \triangleq (\tau - (q' - q)\Delta_t)$ are auxiliary Doppler-shift and time-delay variables, respectively. At integer multiples of auxiliary delay i.e., $\tau_1 = i\Delta_t$ and by using (3) when $c_{m,q} = c_{m',q'}$, the *sinc* function attains its maximum value, resulting in the spike like sidelobes. For detailed explanation, the reader is referred to [25]. With the PSK symbol embedding, the AF in (13) can be re-written as

$$\left| \chi_{\text{FHPSK}}(\tau, \nu, f_{sp}, f'_{sp}) \right| = \left| \sum_{m=1}^M \sum_{m'=1}^M \sum_{q=1}^Q \sum_{q'=1}^Q \chi_{m,m'}^{(\Delta_t)}(\tau, \nu) e^{j(\Omega_{(m,q)} - \Omega_{(m',q')})} e^{j2\pi(f_{sp}m - f'_{sp}m')\gamma} \right|. \quad (16)$$

When $c_{m,q} = c_{m',q'}$ and at $i\Delta_t$ delays, the randomness of the PSK symbols leads to term cancellations inside the summation which reduces the SLL spikes which are exhibited by the FH MIMO radar when there is re-use of FH coefficients [25].

For the special case where FH coefficients are used only once over the PRI, there will be minimum correlation between the FH sub-pulses. In this case, the PSK symbols do not have a pronounced effect on the AF and it would be similar to that of the AF of the FH MIMO radar system.

B. Power Spectral Density analysis

In this section, we analyze the power spectral density (PSD) of both the FH radar and FH/PSK DFRC waveforms.

The PSD of the FH radar waveforms from m^{th} antenna can be expressed as

$$S_{\text{radar}}(f) = \left| \Delta_t \sum_{q=1}^Q \text{sinc}(\Delta_t(2\pi(f - c_{m,q}\Delta_f))) e^{(-j2\pi f q \Delta_t)} \right|^2 \quad (17)$$

From (17), it can be observed that there is a phase term in the PSD expression of the radar waveforms. This phase is linear and, at integer multiples of frequency, $e^{(-j2\pi f q \Delta_t)} = 1$.

The PSD of the m^{th} FH/PSK DFRC waveform can be expressed as

$$S_{\text{FHPSK}}(f) = \left| \Delta_t \sum_{q=1}^Q \text{sinc}(\Delta_t(2\pi(f - c_{m,q}\Delta_f))) e^{-j(2\pi f q \Delta_t - \Omega_{(m,q)})} \right|^2. \quad (18)$$

From (18) we observe that there is an additional phase term ($e^{j\Omega_{(m,q)}}$) involved in the computation of the PSD. This additional phase term disrupts the continuous phase of the FH waveforms. Since the instantaneous frequency is the time dependent derivative of the phase, the system would require broader bandwidth to support this phase discontinuity. This results in the spectrum leakage into the side bands and also an increase in the spectral sidelobe levels.

Also, the amplitude of $S_{\text{FHPSK}}(f)$ depends on the phasor ($e^{j\Omega_{(m,q)}}$) in the summation. When there is reuse of FH coefficients, due to the random nature of $\Omega_{(m,q)}$, the exponential term inside the summation in (18) takes random form. As such, the summation in (18) would result in reduction of the amplitude in the PSD. It is noted that the variation in amplitude and phase makes it difficult for the eavesdropper to intercept the FH pattern.

To avoid the spectral leakage and to confine the spectral contents of the DFRC waveforms to the available bandwidth, we consider a modulation technique where, the higher and lower frequency hops in the bandwidth are left unmodulated. This can be achieved by forcing the phase of the PSK symbols at lower and higher frequencies of the frequency hops to be equal to '0' degrees. This makes the phase at those particular hops continuous in (18), thereby confining the spectral contents of the system to the operational bandwidth. By implementing this technique, the spectral sidelobes also decrease.

C. Effect of the partial phase modulation technique on the AF

The radar benefits from the PSK embedding due to the reduction in the SLL of the AF. This is due to the phasor nature of the exponential in (16). The PSK symbols are generated in a random fashion and hence the reduction in the SLL. Therefore, when we limit the spectral contents to the allocated bandwidth by forcing a zero phase at the lower and higher frequencies of the bandwidth, the randomness in the phase values decreases and these phase modulated hops will be similar to that of the MIMO radar frequency hops. In this case, the *sinc* function in the AF computation in (15) takes the maximum value and the phase difference term $e^{j(\Omega_{(m,q)} - \Omega_{(m',q')})} = 1$. Hence the AF of MIMO DFRC system will be similar to that of the AF of FH MIMO radar system at a given integer delay. This leads to slight increase in the SLL of the AF of the DFRC system. But the phase modulation of the remaining hops still yield lower SLL compared to the AF of the FH MIMO radar system.

There is also an exception where these zero phase values can reduce the SLL of the AF. This happens when the combination of the random phases ($\Omega_{(m,q)}$) takes the same phase value other than '0' over all the hops. In this case,

$e^{j(\Omega_{(m,q)} - \Omega_{(m',q')})} \neq 1$ for few terms in the summation and the exponential term takes unit complex values. Therefore, there will be slight reduction in the SLL.

IV. COMMUNICATION SYMBOL EMBEDDING WITH PHASE CONTINUITY

A. Waveform Design and Analysis

In order to maintain the phase continuity between the successive frequency hops, modulation of the FH waveforms with frequency shift keying (FSK) symbols for the DFRC operation is considered in this section. The FSK signals have constant envelope and are easy to implement. They are generated by modulating the frequency of the carrier signal to represent the bits. The frequencies are selected as integer multiples of the bit rate and the oscillators are considered to have the same initial phase during the frequency generation to maintain the phase continuity between the successive symbols. In this paper, we consider a case of binary FSK (BFSK), where the binary '0' is represented by frequency f_L and the binary '1' is represented by frequency f_H [30]. The BFSK modulated FH waveforms from m^{th} antenna can be expressed as

$$\rho_m(t, n) = \sum_{q=1}^Q e^{j(2\pi f_{(m,q)}t)} h_{m,q}(t) u(t - q\Delta_t - nT_0), \quad (19)$$

where, $f_{m,q} \in \{f_L, f_H\}$. Using the AF expression for the MIMO signal model from (12), the AF of the FH/FSK system can be expressed as

$$|\chi_{\text{FHFSK}}(\tau, \nu, f_{sp}, f'_{sp})| = \left| \sum_{m=1}^M \sum_{m'=1}^M \sum_{q=1}^Q \sum_{q'=1}^Q \chi_{m,m'}^{(\text{FSK})}(\tau, \nu) e^{j2\pi(f_{sp}m - f'_{sp}m')\tau} \right|, \quad (20)$$

where,

$$\chi_{m,m'}^{(\text{FSK})}(\tau, \nu) = \chi_{m,m'}^{\text{rect}}(\tau_2, \nu_2) e^{j2\pi(c_{m',q'}\Delta_f + f_{(m',q')}\tau)}, \quad (21)$$

$$\begin{aligned} \chi_{m,m'}^{\text{rect}}(\tau_2, \nu_2) &\triangleq \int_0^{\Delta_t} u(t)u(t + \tau_2) e^{j2\pi\nu_2 t} dt, \quad |\tau_2| \leq \Delta_t \\ &= \frac{\Delta_t - |\tau_2|}{\Delta_t} \text{sinc}(\nu_2(\Delta_t - |\tau_2|)) e^{j\pi\nu_2(\Delta_t - |\tau_2|)}. \end{aligned} \quad (22)$$

In (21), $\nu_2 \triangleq (c_{m,q} - c_{m',q'})\Delta_f + (f_{(m,q)} - f_{(m',q')}) + \nu$ and $\tau_2 \triangleq (\tau - (q' - q)\Delta_t)$ are auxiliary Doppler-shift and time-delay variables for FH/FSK waveforms respectively.

From (22), it is evident that the FH/FSK AF depends on the *sinc* function. At integer values of auxiliary delay and zero Doppler shift, when $(c_{m,q} = c_{m',q'})$, the AF depends on the frequency shifts $f_{(m,q)}$ and $f_{(m',q')}$. When $f_{(m,q)} = f_{(m',q')}$, the AF of FH/FSK will be similar to that of FH MIMO radar. But, when $f_{(m,q)} \neq f_{(m',q')}$, the *sinc* function equals to '0' thereby reducing the SLL of the AF. However, there is a case where the SLL of the FH/FSK system are greater than the FH radar system. When $(c_{m,q} \neq c_{m',q'})$ and $f_{(m,q)} \neq f_{(m',q')}$ but the value of $((c_{m,q} - c_{m',q'})\Delta_f + f_{(m,q)} - f_{(m',q')}) = 0$, the

sinc function attains its maximum value. This value of the AF SLLs is always less than the absolute maximum value that is achieved by the FH MIMO radar system. The PSD of the m^{th} FH/FSK waveform can be expressed as

$$S_{\text{FHFSK}}(f) = \Delta_t \sum_{q=1}^Q \left| \text{sinc}(\Delta_t(2\pi(f - c_{m,q}\Delta_f - f_{(m,q)}))) e^{-j(2\pi f q \Delta_t)} \right|^2. \quad (23)$$

From (23), it is evident that there is a shift in the PSD of the DFRC system using FH/FSK waveforms. To prevent leakage into adjacent frequency bands, the lower and higher frequency hops of the bandwidth are left unmodulated.

B. Comparison between FH/PSK and FH/FSK

The performance of FSK and PSK communication symbol embedding for the DFRC platforms is compared in this section.

From radar perspective, bandwidth is the important criteria for the waveform design. The PSK symbols have spectral leakage and the FSK signals produce a shift in the frequencies when embedded into the FH radar waveforms. This can be avoided by not modulating the lower and higher frequency hops in a given bandwidth for both cases. The AF SLL are also important for the radar for detection purposes. Both the systems exhibit reduction in the SLL. But the SLL reduction in FH/FSK is not as pronounced as FH/PSK. There are also some cases where, FH/FSK system exhibit even higher SLL than the FH radar system.

From communication system perspective, data rate and detection of symbols is important. For a given bandwidth, the PSK symbol embedding can achieve higher data rate compared to that of FSK. When it comes to detection, PSK symbol detection requires phase synchronization resulting in a more complex symbol detector. Whereas the FSK symbols can be simply detected by estimating the shift in the frequency.

V. SIMULATIONS

We consider a MIMO radar system operating at X-band with carrier frequency $f_c = 8.2$ GHz and bandwidth 100 MHz. The sampling frequency is taken as the Nyquist rate, i.e., $f_s = 2 \times 10^8$ sample/sec. The PRI is $T_0 = 10 \mu\text{s}$, i.e., the PRF is 100 KHz. The transmit array comprises $M = 10$ omnidirectional transmit antennas spaced half a wavelength apart. We generate a set of 16 FH waveforms. The parameter $K = 16$ is chosen such that the FH step is $\Delta_f = 6$ MHz. The FH code length $Q = 15$ is assumed and the FH interval duration $\Delta_t = 0.167 \mu\text{s}$ is used. The 10×15 FH code is generated randomly from the set $\{1, 2, \dots, K\}$, where $K = 16$.

From Fig. 1a, we can observe that the disruption in the continuous phase of the FH waveforms due to PSK symbol embedding results in spectral leakage into the side bands and increase in the spectral sidelobe levels. It is evident from Fig. 1b that the partial modulation of phases confines the spectral contents of the system to the operating bandwidth. As a result

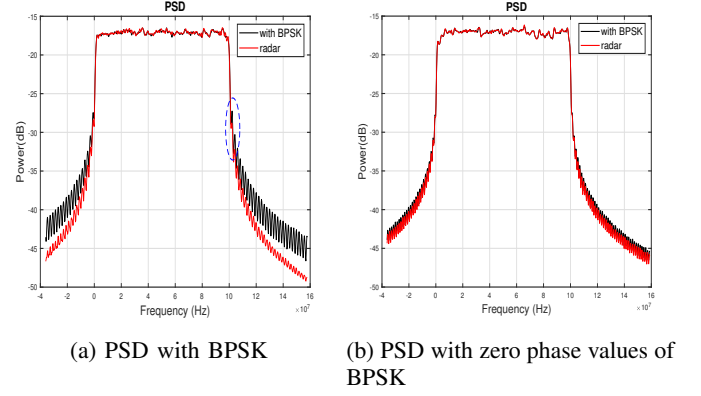


Fig. 1: PSD of FH radar and FH/PSK DFRC systems

of this modulation, it can be observed from 2a and Fig. 2b that the range sidelobe levels of the FH/PSK MIMO DFRC system are increased. FSK information embedding into the FH

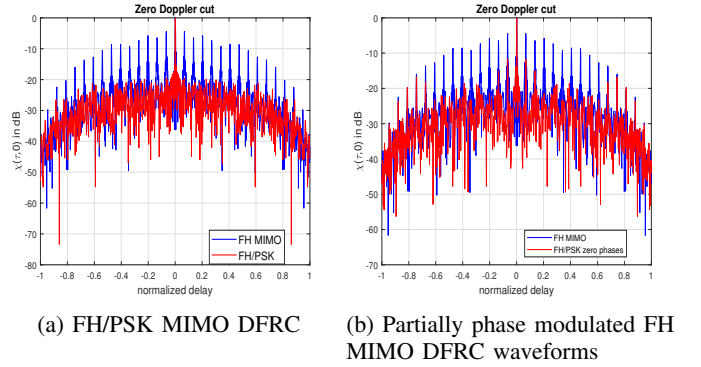


Fig. 2: Zero Doppler cuts for FH/PSK DFRC system

MIMO waveforms preserves the continuous phase between the successive frequency hops but there is a frequency shift due to the change of hop frequencies which can be observed in Fig. 3a. By leaving the higher and lower end frequencies of the bandwidth of the FH MIMO radar waveforms unmodulated, the spectral components of the FH/FSK DFRC system can be confined to the operational bandwidth, as shown in Fig. 3b. It

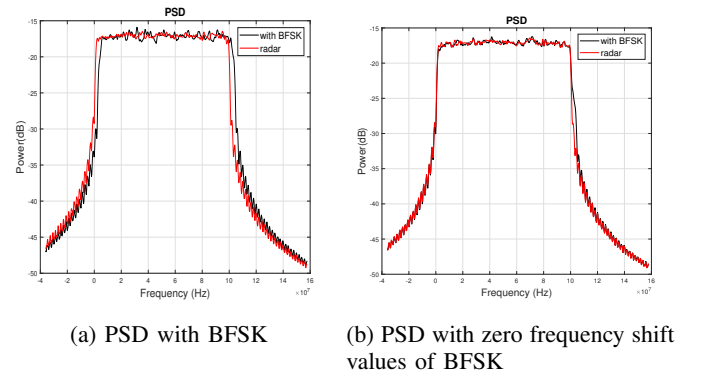


Fig. 3: PSD of FH radar and FH/FSK DFRC systems

is also evident from Fig. 4 that information embedding into the FH MIMO waveforms using PSK or FSK symbols break the strong correlations between the FH sub-pulses, which in

turn reduces the range sidelobe levels of the FH MIMO DFRC system.

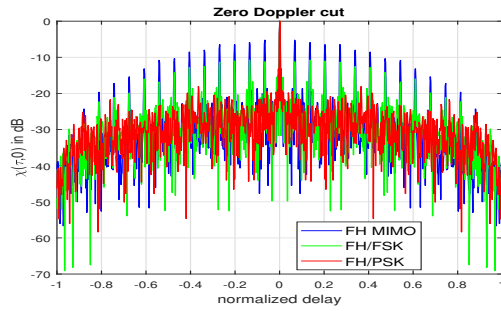


Fig. 4: Zero Doppler cuts for the FH MIMO radar and FH/BFSK systems

VI. CONCLUSION

This paper examined the effects of phase and frequency modulations on FH radar waveforms in DFRC platforms. The random nature of the communication alphabets breaks any strong correlations among similar hops, leading to significant reduction in range and Doppler sidelobes. On the other hand, the phase discontinuity introduced by the modulation of FH waveforms with PSK symbols causes spectral leakage into sidebands and increases spectrum sidelobe levels. Using FSK symbols, in lieu of PSK symbols, provided phase continuity, but presented a frequency shift owing to changes in hop frequencies by the communication signal. To avoid, or significantly reduce, the spectral leakage and frequency shifts, a solution of partial modulation of the frequency hop pulses was considered, and the trade-off between the spectral leakage and AF SLL was delineated. Our analysis and results showed that the DFRC system has lower SLL with both phase and frequency modulation compared to nominal FH MIMO radar system. Further analysis is required to examine the FH/PSK and FH/FSK MIMO DFRC system performance in presence of clutter and multipath channels for communications.

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